THINKING QUANTUM: A NEW PERSPECTIVE ON DECISIONMAKING IN LAW

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ABSTRACT

Behavioral law and economics (BLE) is built on the observation that human decisionmaking is often incompatible with rational choice theory. But if our decisions do not follow the rules of rational choice theory, is there a general set of rules they do follow? Can BLE offer a coherent alternative to rational choice?

To the extent it has addressed these questions, BLE has struggled with them. But an emerging psychological theory called "quantum decisionmaking" may offer answers. Quantum decisionmaking offers a new perspective on how people think about probabilities—one that challenges core assumptions of rational choice theory. Specifically, quantum decisionmaking assumes that probabilistic judgments are prone to systematic path dependencies. Your next judgment is apt to be influenced by your last judgment, which was likely influenced by the one before that. By incorporating such dependencies, quantum models of decisionmaking can account for classically "rational" decisions and a variety of the heuristics and biases that animate BLE.

Quantum decisionmaking has theoretical and practical implications for law. On a theoretical level, quantum decisionmaking conceptually unifies what has sometimes been characterized as BLE's ad hoc list of heuristics and biases. More practically, quantum decisionmaking highlights the important role that sequence plays in law's choice architecture, and generates new, testable predictions about a variety of important law-related decisions. We identify and explore eight such predictions, which concern issues ranging from juror decisionmaking to witness lineups to policing.

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** Assistant Professor of Psychology, Vanderbilt University. Jennifer S. Trueblood would like to acknowledge support received from National Science Foundation grants SES-1556415 and SES-1556325.

I. INTRODUCTION

Law is often informed, explicitly or implicitly, by theories of judgment[1] and decisionmaking[2] developed in the social sciences.[3] Rational choice theory has been particularly influential.[4]

Rational choice theory posits that people "rationally maximize" their individual expected utilities:[5] Decision makers act as though[6]

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1. In this Article, we use the word "judgment" to refer to a probabilistic determination (e.g., what is the likelihood that it will rain this afternoon?).

2. In this Article, we use the word "decision" to refer to a choice among alternatives (e.g., will you carry an umbrella today or not?). Decisions are, of course, related to judgments—your decision about carrying an umbrella will be informed by your judgment about the likelihood of rain.

3. *E.g.*, Christine Jolls, Cass R. Sunstein & Richard Thaler, *A Behavioral Approach to Law and Economics*, 50 STAN. L. REV. 1471, 1474 (1998) (contending that behavioral research in judgment and decisionmaking can and should shape law and policy); Owen D. Jones, *Why Behavioral Economics Isn't Better, and How It Could Be*, *in* RESEARCH HANDBOOK ON BEHAVIORAL LAW AND ECONOMICS 476, 481 (J. Titelbaum & K. Zelier eds., 2015) ("Law is fundamentally a consumer of behavioral models."); Donald C. Langevoort, *Behavioral Theories of Judgment and Decision Making in Legal Scholarship: A Literature Review*, 51 VAND. L. REV. 1499, 1499 (1998) ("Nearly all interesting legal issues require accurate predictions about human behavior to be resolved satisfactorily."); Jeffrey J. Rachlinski, *The "New" Law and Psychology: A Reply to Critics, Skeptics, and Cautious Supporters*, 85 CORNELL L. REV. 739, 739 (2000) ("In order to understand how the law has developed and how law should develop, scholars must be able to predict people's responses to legal rules accurately."); Dan Simon, *A Third View from the Black Box: Cognitive Coherence in Legal Decision Making*, 71 U. CHI. L. REV. 511, 513 (2004) (arguing that if we understand the cognitive phenomena behind decisionmaking, "we can devise interventions and introduce procedures that reduce the risk of error and thus make the decision-making process better fit the legal ideals it is intended to serve.").

4. *See, e.g.*, Robert D. Cooter & Daniel L. Rubinfeld, *Economic Analysis of Legal Disputes and Their Resolution*, 27 J. ECON. LITERATURE 1067, 1068 (1989) (describing the widespread influence of economic analysis in law); Robert A. Hillman, *The Limits of Behavioral Decision Theory in Legal Analysis: The Case of Liquidated Damages*, 85 CORNELL L. REV. 717, 717 (2009) ("Economic analysis of law is, of course, the predominant example of legal analysts' turn to social science."); Richard A. Posner, *The Law and Economics Movement*, 77 AM. ECON. REV. 1, 1 (1987) (noting broad influences of economic theory on law). For a thorough discussion of what "rational choice theory is and is not" and the variants of rational choice theory relied upon in law, see Russell B. Korobkin & Thomas S. Ulen, *Law and Behavioral Science: Removing the Rationality Assumption From Law and Economics*, 88 CAL. L. REV. 1051, 1055-66 (2000).

5. *E.g.*, RICHARD A. POSNER, ECONOMIC ANALYSIS OF LAW 3 (4th ed. 1992); *see also* Richard A. Posner, *Are We One Self or Multiple Selves?: Implications for Law and Public Policy*, 3 LEGAL THEORY 23, 24 (1997) ("Man is a rational maximizer of his ends.").

6. Importantly, we note that economists are generally not concerned with the deliberative processes (if any) that lead people to make a particular choice. Economists are chiefly concerned with behavior—the "output" of decisionmaking—and not with the processes that generate it. *E.g.*, Joshua D. Wright & Douglas H. Ginsburg, *Behavioral Law and Economics: Its Origins, Fatal Flaws, and Implications for Liberty*, 106 NW. U. L. REV. 1033, 1037 (2015).

they gather the optimal amount of available information about decision alternatives,[7] use that information to judge the probabilities of potential outcomes in accordance with the rules of classical (Bayesian) probability theory,[8] and use those probability judgments (in combination with their individual preferences) to select the alternative that offers the greatest expected utility.[9] Rational choice theory provides the foundation of the law and economics movement,[10] which, in turn,

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7. GARY S. BECKER, THE ECONOMIC APPROACH TO HUMAN BEHAVIOR 14 (1976) (noting that decision makers are assumed to "accumulate an optimal amount of information and other inputs in a variety of markets"); Russell Korobkin, *A Traditional and Behavioral Law-and-Economics Analysis of Williams v. Walker-Thomas Furniture Company*, 26 U. HAW. L. REV. 441, 447 (2004) ("[M]ost versions of [rational choice theory] assume, at a minimum, that individuals will use all available information to select behaviors that maximize their expected utility.").

8. Peter D. Bruza, Zheng Wang & Jerome R. Busemeyer, *Quantum Cognition: A New Theoretical Approach to Psychology*, 19 TRENDS IN COGNITIVE SCI. 383, 383 (2015) ("Although rational models of cognition have become prominent and have achieved much success, they adhere to the laws of classical probability theory . . . ."); Robert C. Ellickson, *Bringing Culture and Human Frailty to Rational Actors: A Critique of Classical Law and Economics*, 65 CHI.-KENT L. REV. 23, 23 (1989) ("The [rational choice] model assumes that a person can perfectly process available information about alternative courses of action . . . ."); Michael S. Pardo, *Introduction to the Meador Lectures on Rationality*, 64 ALA. L. REV. 141, 144 n.15 (2012) (noting the probability component of expected utility judgments relies on "axioms of classical probability theory"); Alex Stein, *The Flawed Probabilistic Foundation of Law and Economics*, 105 NW. U. L. REV. 199, 200-02, 254 (2011) (noting that the law and economics literature assumes that the issue of how rational actors think about probability is settled in favor of what Professor Stein calls the "axiomatized view of probability," which is Bayesian); Charles Yablon, *The Meaning of Probability Judgments: An Essay on the Use and Misuse of Behavioral Economics*, 2004 U. ILL. L. REV. 899, 934 ("Behavioral theory first entered scholarly legal debates as an empirical rebuttal to rational choice models, which assumed, usually on little or no empirical evidence, that individuals could accurately assess risk and other probabilistic concepts, or that errors in such probability assessments would be randomly distributed and could therefore be ignored.").

9. *See* Becker, *supra* note 7 (defining the core features of rational choice theory); *see also* Korobkin & Ulen, *supra* note 4, at 1062-64 (describing the translation of rational choice theory into expected utility models, the "most dominant" conception of rational choice theory in modern microeconomics); Jennifer Arlen, *Comment: The Future of Behavioral Economic Analysis of Law*, 51 VAND. L. REV. 1765, 1766 (1998) ("Conventional law and economics assumes that people exhibit rational choice: that people are self-interested utility maximizers with stable preferences and the capacity to optimally accumulate and assess information.").

10. Korobkin & Ulen, *supra* note 4, at 1055 ("[E]arly law-and-economics scholars imported from economics a series of assumptions about how people respond to incentives, known generally as 'rational choice theory.' "); *see also infra* Part II.

provides the foundation for policies ranging from cap-and-trade environmental regulation to tort reform.[11] Further, rational choice assumptions inform the dominant "rationalist" account of how legal decision makers process evidence and reach verdicts.[12]

Of course, people are not always rational.[13] Thus, many scholars have advocated rethinking law's reliance on rational choice assumptions.[14] Operating largely under the banner of behavioral law and economics (BLE), these scholars have been tremendously successful in identifying specific situations in which people tend to make irrational decisions and developing research-based policy recommendations to "debias," or "nudge," people in those situations.[15] But BLE has also received its share of criticism, much of which focuses on its atheoretical

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11. *See* Robert W. Hahn & Robert N. Stavins, *The Effect of Allowance Allocations on Cap-and-Trade System Performance*, 54 J.L. & ECON. S267, S276 (2011) (identifying the economic justifications for cap-and-trade environmental policies); George L. Priest, *The Monsanto Lectures: Modern Tort Law and Its Reform*, 22 VAL. U. L. REV. 1 (1987) (describing economic bases of tort reform efforts).

12. Kenworthy Bilz, *We Don't Want to Hear It: Psychology, Literature, and the Narrative Model of Judging*, 2010 U. ILL. L. REV. 429, 429 (2010) ("The currently dominant view of how people process evidence and draw conclusions is linear and Bayesian."); Simon, *supra* note 3, at 515 ("The Rationalist view resonates with theories of logic, rational choice models of decision making, and Bayes Theorem.").

13. *See, e.g.*, Daniel Kahneman & Amos Tversky, *On the Psychology of Prediction*, 80 PSYCHOL. REV. 237 (1973) [hereinafter Kahneman & Tversky, *Psychology of Prediction*]; Daniel Kahneman & Amos Tversky, *On the Study of Statistical Intuitions*, *in* JUDGMENTS UNDER UNCERTAINTY: HEURISTICS AND BIASES (Daniel Kahneman, Paul Slovic & Amos Tversky eds., 1982); Herbert A. Simon, *A Behavioral Model of Rational Choice*, 69 Q. J. ECON. 99 (1955); Amos Tversky & Daniel Kahneman, *Evidential Impact of Base Rates*, *in* JUDGMENTS UNDER UNCERTAINTY: HEURISTICS AND BIASES (Daniel Kahneman, Paul Slovic & Amos Tversky, eds., 1982); Amos Tversky & Daniel Kahneman, *Extensional Versus Intuitive Reasoning: The Conjunction Fallacy in Probability Judgments*, 90 PSYCHOL. REV. 293 (1983) [hereinafter Tversky & Kahneman, *The Conjunction Fallacy*]; Amos Tversky & Daniel Kahneman, *Judgment Under Uncertainty: Heuristics and Biases*, 185 SCIENCE 1124 (1974) [hereinafter Tversky & Kahneman, *Judgment Under Uncertainty*]. For more general discussion of psychological research on "irrational" decisionmaking and its relationship to behavioral economics, see Colin Camerer, *Behavioral Economics: Reunifying Psychology and Economics*, 96 PROC. NAT'L. ACAD. SCI. 10575 (1999); David Laibson & Richard Zeckhauser, *Amos Tversky and the Ascent of Behavioral Economics*, 16 J. RISK AND UNCERTAINTY 7 (1998).

14. *See, e.g.*, Jolls, Sunstein & Thaler, *supra* note 3, at 1476-81 (identifying the task of BLE and discussing bounded rationality, bounded willpower, and bounded self-interest, in turn). Professor Raj Chetty describes a trend in recent decades toward formalizing "the implications of psychology for economics . . . using mathematical models . . . ." Raj Chetty, *Behavioral Economics and Public Policy: A Pragmatic Perspective*, 105 AM. ECON. REV. 1, 1 n.1 (2015).

15. *See, e.g.*, Jolls, Sunstein & Thaler, *supra* note 3, at 1523-41 (offering research-based prescriptions for a variety of legal issues); Rachlinski, *supra* note 3, at 760-63 (using the issue of treatment of liquidated damages as a test case for application of BLE, in response to Hillman, *supra* note 4); RICHARD H. THALER & CASS R. SUNSTEIN, NUDGE: IMPROVING DECISIONS ABOUT HEALTH, WEALTH, AND HAPPINESS (2009); Jeffrey J. Rachlinski, *Gains, Losses, and the Psychology of Litigation*, 70 S. CAL. L. REV. 113, 167-76 (1996) (discussing implications of behavioral data for the settlement of disputes).

nature.[16] Although BLE's proponents can point to a hodge-podge of specific situations in which behavior violates the assumptions of rational choice theory, the critique goes, they fail to offer any cohesive account of these violations—they offer no general alternative to rational choice.[17] While rational choice theory may be imperfect, it remains a more useful tool for guiding the law than an ad hoc list of exceptions.[18]

The exchange between the law and economics camp and the BLE camp has fueled, and continues to fuel, valuable legal scholarship.[19] But some relevant developments in the psychological research on judgment and decisionmaking have gone largely unnoticed in the legal literature. This Article focuses on one such development—the emergence

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16. *See* Richard A. Posner, *Rational Choice, Behavioral Economics, and the Law*, 50 STAN. L. REV. 1551, 1552 (1998) (characterizing the behavioral law and economics approach as "ad hoc" and "antitheoretical"); Arlen, *supra* note 9, at 1777 (1998) (critiquing behavioral law and economics for failing to provide a coherent alternative model of human behavior). For a good discussion of this critique, see Rachlinksi, *supra* note 3, at 748-52 (discussing, and responding to, the "laundry list" critique of behavioral law and economics). For a more general critique of law's use of psychological research as atheoretical, see Jeremy A. Blumenthal, *Law and Social Science in the Twenty-First Century*, 12 S. CAL. INTERDISC. L.J. 1, 31-32 (2002) (discussing a perceived lack of theoretical sophistication in legal psychological work); Mark A. Small, *Legal Psychology and Therapeutic Jurisprudence*, 37 ST. LOUIS U. L.J. 675, 690-92 (1993) (reviewing a sample of law and psychology articles and concluding that the overwhelming majority were purely descriptive and lacked theoretical sophistication).

17. *See* Mark Kelman, *Behavioral Economics as Part of a Rhetorical Duet: A Response to Jolls, Sunstein, and Thaler*, 50 STAN. L. REV. 1577, 1586 (1998) (asserting that behavioral law and economics seems "to confuse discordant observations for a countertheory and evade questions about the gaps in the behavioral picture. . . "); Arlen, *supra* note 16, at 1777 ("Behavioral economic analysis of law cannot serve as the basis for broad normative policy conclusions because it cannot provide a coherent alternative model of human behavior capable of generating testable predictions and policy conclusions in a wide range of areas."); Chetty, *supra* note 14, at 32 ("A common criticism of behavioral economics is that it does not offer a single unified framework as an alternative to the neoclassical model."). Indeed, even scholars within the behavioral law and economics movement have acknowledged that it "lacks a single, coherent theory," and that a "general theory" would be welcome. Korobkin & Ulen, *supra* note 4, at 1057.

18. Kelman, *supra* note 17, at 1586 ("[B]ehavioral economics can better be seen as a series of particular counterstories, formed largely in parasitic reaction to the unduly self-confident predictions of rational choice theorists, than as an alternative general theory of human behavior."); *cf.* Cass R. Sunstein, *Behavioral Law and Economics: A Progress Report*, 1 AM. L. & ECON. REV. 115, 147 (1999) (acknowledging that some object to BLE on the grounds that it is "better to work with the simpler tools of conventional economics, just because of their simplicity"). The argument, in short, is that it "takes a theory to beat a theory." Jeffrey J. Rachlinksi, *The Psychological Foundations of Behavioral Law and Economics*, 2011 U. ILL. L. REV. 1675, 1687 (2011).

19. *See generally* Avishalom Tor, *The Next Generation of Behavioral Law and Economics*, *in* EUROPEAN PERSPECTIVES ON BEHAVIOURAL LAW AND ECONOMICS 17 (Klaus Mathis ed., 2015) (noting that Jolls, Sunstein & Thaler, *supra* note 3, was (as of 2011) among the 100 most-cited law review articles of all time); Blumenthal, *supra* note 16, at 1 ("The use of social science—of psychology in particular—to inform legal theory and practice is fast becoming the latest craze in the pages of legal academia.").

of a theory of quantum decisionmaking—and explores some of its implications for BLE and for the law more generally.

The idea behind quantum decisionmaking is that, although the "heuristics and biases" underlying BLE are incompatible with the rules of rational choice theory, they are not anomalies.[20] Rather, they are natural products of a different, more-comprehensive set of decisionmaking rules.[21] These rules differ from the rules posited by rational choice theory in one crucial respect: while rational choice theory assumes that decision makers judge probabilities in accordance with *classical probability theory*,[22] quantum decisionmaking assumes that decision makers judge probabilities in accordance with *quantum probability theory*.[23]

This may initially strike the reader as strange. "Quantum" is a physics term, not a psychology term, and quantum probability theory

20. *See, e.g.*, Emmanuel M. Pothos & Jerome R. Busemeyer, *Can Quantum Probability Provide a New Direction for Cognitive Modeling?*, 36 BEHAV. & BRAIN SCI. 255 (2013) (arguing that while it is difficult to account for heuristics and biases with classical, rationalist models of decisionmaking, "these same findings have natural and straightforward explanations with quantum principles."); James M. Yearsley & Jennifer S. Trueblood, *A Quantum Theory Account of Order Effects and Conjunction Fallacies in Political Judgments*, 25 PSYCHONOMIC BULL. & REV. 1517, 1517 (2018) ("One key advantage of using quantum theory [as a modeling framework] is that it explains multiple types of judgment errors using the same basic machinery, unifying what have previously been thought of as disparate phenomena.").

21. *See* Mehrdad Ashtiani & Mohammad Abdollahi Azogmi, *A Survey of Quantum-like Approaches to Decision Making and Cognition*, 75 MATHEMATICAL SOC. SCI. 49, 49 (2015) (abstract) ("Although [rational choice theory and expected utility theory] provide a suitable ground for modeling the decision making process of humans, they are unable to explain the corresponding irrationalities and existing paradoxes and fallacies. Recently, a new [quantum] formulation of decision theory that can correctly describe these paradoxes and possibly provide a unified and general theory of decision making has been proposed."); Zheng Wang et al., *Context Effects Produced by Question Orders Reveal Quantum Nature of Human Judgments*, 111 PROC. NAT'L. ACAD. SCI. 9431, 9431 (2014) ("Recently, a group of psychologists and physicists have formulated new rules for human reasoning under uncertainty based on quantum probability theory."); Bruza, Wang & Busemeyer, *supra* note 8, at 387 ("Rather than resorting to heuristics, quantum cognition successfully accounts for these violations using a coherent, common set of principles."). The authoritative book that began laying out these rules is: JEROME R. BUSEMEYER & PETER D. BRUZA, QUANTUM MODELS OF COGNITION AND DECISION (2012).

22. Bruza, Wang & Busemeyer, *supra* note 8, at 383 ("Although rational models of cognition have become prominent and have achieved much success, they adhere to the laws of classical probability theory . . . .").

23. We provide an introductory-level description of quantum decisionmaking for a legal audience in *infra* Part III. For a general introduction to quantum decisionmaking in the psychological literature, see Jerome R. Busemeyer & Zheng Wang, *What is Quantum Cognition, and How is it Applied to Psychology?*, 24 CURRENT DIRECTIONS IN PSYCHOL. SCI. 163 (2015); Bruza, Wang & Busemeyer, *supra* note 8.

typically relates to particle physics, not human decisions.[24] But decisionmaking may be more analogous to particle physics than the reader expects.[25] Both human decisions and particle measurements are influenced by forms of path dependency.[26] The judgments we make are influenced by the judgments that came before them, just as measurements of subatomic particles are influenced by the measurements that came before them.[27] In the context of particle measurements, these path dependencies are systematic: Physicists developed quantum probability theory to account for the systematic influences of measurements on future measurements.[28] Quantum decisionmaking posits that path dependencies in human judgment are analogously systematic, and that quantum probability theory can account for systematic influences of judgments on future judgments.[29]

In a growing number of psychological studies, quantum models of decisionmaking predict people's decisions better than their rational choice counterparts,[30] accounting for many of the heuristics and biases underlying the BLE movement.[31] Quantum decisionmaking, it seems,

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24. Merriam-Webster, for example, provides a physics definition for "quantum:" "any of the very small increments or parcels into which many forms of energy are subdivided." *Quantum*, MERRIAM-WEBSTER ONLINE DICTIONARY, https://www.merriam-webster.com/dictionary/ quantum (last visited Jan. 3, 2017).

25. New insights into the operations of the physical world have traditionally informed new perspectives in both the social sciences and in law. *See, e.g.*, ALEXANDER WENDT, QUANTUM MIND AND SOCIAL SCIENCE: UNIFYING PHYSICAL AND SOCIAL ONTOLOGY 4 (2015) (observing that many of the statistical methods used by social scientists were based on classical probability theory, which "came from the *previous*, Newtonian revolution in physics . . . ."); William Bennett Munro, *Physics and Politics—An Old Analogy Revised*, 22 AM. POL. SCI. REV. 1, 1 (1928) (echoing Walter Bagehot's observation that "the advance in natural science seemed to suggest modifications in the old theories of the state and of government."); Lawrence H. Tribe, *The Curvature of Constitutional Space: What Lawyers Can Learn from Modern Physics*, 103 HARV. L. REV. 1, 2 (1989) (observing that the "metaphors and intuitions that guide physicists can enrich our comprehension of social and legal issues," and that thinking about legal institutions "has been fundamentally influenced by new insights into the operation of the physical world.").

26. Jerome R. Busemeyer et al., *A Quantum Theoretical Explanation for Probability Judgment Errors*, 118 PSYCHOL. REV. 193, 193 (2011).

27. *Id.*

28. Jennifer S. Trueblood & Jerome R. Busemeyer, *Quantum Information Processing Theory*, *in* ENCYCLOPEDIA OF THE SCIENCES OF LEARNING (2012) (describing issues with contextual influence that "faced physicists in the 1920s that forced them to develop quantum theory").

29. *Id.*; *see also* Pothos & Busemeyer, *supra* note 20, at 256 ("[Quantum probability] theory is, in principle, applicable not just in physics, but in any science in which there is a need to formalize uncertainty.") For discussion, see *infra* Part III.

30. In this Article, we use the term "theory" to refer to a set of general principles, and we use the term "model" to refer to a specific application of those principles to a situation (sometimes expressed in mathematical terms). For example, rational choice theory is a general set of principles that can be incorporated in specific models of various decisions, from choosing a candy bar to reaching a verdict in a murder trial.

31. *See, e.g.*, Jerome R. Busemeyer et al., *The Conjunction Fallacy, Confirmation, and Quantum Theory: Comment on Tentori, Crupi, and Russo (2013)*, 144 J. EXP. PSYCHOL.: GEN.

captures something about human judgment and decisionmaking that rational choice theory does not.[32]

This has substantial theoretical and practical implications for law. On a theoretical level, quantum decisionmaking offers an answer to the critique that BLE is ad hoc—and, potentially, the "general theory" that BLE has been said to both need and lack[33] (though we stress that important steps remain to be taken).[34] On a practical level, quantum decisionmaking can and has generated new, testable hypotheses about judgment and decisionmaking that are relevant in a variety of legal contexts.[35] Indeed, the central idea of quantum decisionmaking—that judgments systematically influence subsequent judgments—is inherently relevant in essentially any legal situation that requires multiple judgments to be made in succession. From jurors deciding cases that involve multiple claims to witnesses choosing perpetrators from

236 (2015); Riccardo Franco, *Quantum Amplitude Amplification Algorithm: An Explanation of Availability Bias*, *in* QUANTUM INTERACTION (Peter Bruza et al. eds., 2009); Emmanuel M. Pothos & Jerome R. Busemeyer, *A Quantum Probability Explanation for Violations of 'Rational' Decision Theory*, 276 PROC. R. SOC. B 2171 (2009); Jennifer S. Trueblood & Jerome R. Busemeyer, *A Quantum Probability Account of Order Effects in Inference*, 35 COG. SCI. 1518 (2011); Zheng Wang & Jerome R. Busemeyer, *A Quantum Question Order Model Supported by Empirical Tests of an A Priori and Precise Prediction*, 5 TOPICS IN COGNITIVE SCI. 689 (2013); Busemeyer et al., *supra* note 26. *See generally* WENDT, *supra* note 27 (noting the "growing experimental evidence that long-standing anomalies of human behavior can be predicted by 'quantum decision theory.' ").

32. As explained below, however, quantum decisionmaking researchers make no claim at the present time about the physical basis of this "something." *See* Busemeyer et al., *supra* note 26, at 193 (the researchers state that they do not claim that the brain physically functions like a quantum computer; they use the quantum analogy only to predict decisions).

33. Some researchers have explicitly posited that quantum decision may provide a framework for bounded rationality. *See* Jerome R. Busemeyer & Jennifer S. Trueblood, *Theoretical and Empirical Reasons for Considering the Application of Quantum Probability Theory*, *in* HUMAN COGNITION IN PROCEEDINGS OF THE QUANTUM COGNITION MEETS TARK, WORKSHOP, GRONINGEN, THE NETHERLANDS (2011), http://www.ai.rug.nl/conf/quantumTARK/busemeyer.pdf [https://perma.cc/HUL8-CBJ2]; Emmanuel M. Pothos & Jerome R. Busemeyer, *In Search for a Standard of Rationality*, 5 FRONTIERS IN PSYCH. 1,2 (2014) (quantum probability theory "is perhaps a framework for bounded rationality: Perhaps not as rational as in principle possible (assuming [classical probability] theory is the ultimate standard of rationality), but the best that can be achieved, given (broadly assumed) limitations in the representational capacity of the cognitive system."). "Quantum theory provides a simple account that unifies all of the diverse findings [about human judgment and decisionmaking behavior] within a common theoretical framework." In fact, quantum decisionmaking even predicts some previously unnoticed patterns in those irrationalities. *See, e.g.*, Wang et al., *supra* note 21 (predicting, based on quantum principles, a pattern called "QQ equality" in the order effects that participants displayed).

34. For discussion of some of these steps, see *infra* Part V.

35. *See, e.g.*, James M. Yearsley & Emmanuel M. Pothos, *Zeno's Paradox in Decisionmaking*, 283 PROC. ROYAL SOC'Y B, at 1,2 (2016), https://royalsocietypublishing.org/doi/pdf/10.1098/rspb.2016.0291 [https://perma.cc/7EDT-G78K] (generating and testing a prediction about the "quantum Zeno effect"); Wang & Busemeyer, *supra* note 31 (generating and testing an *a priori* prediction about order effects); *see generally infra* Part IV.B.

lineups to employees filling out investment forms, quantum decisionmaking illuminates the importance of sequence in law's choice architecture.[36]

This Article introduces quantum decisionmaking to a legal readership, discusses some of its implications for the law, and explores its potential to inform policy. Part II of this Article situates quantum decisionmaking within the existing body of legal scholarship concerned with judgment and decisionmaking. Part II largely focuses on law and economics, BLE, and the tension between the two approaches, while also briefly discussing several "constructivist" models of juror decisionmaking. Part III then introduces quantum decisionmaking. Part III.A describes key features of quantum decisionmaking on a conceptual level. Part III.B provides an example of a quantum model of a legal judgment, illustrating how that model predicts several heuristics and biases.[37]

Part IV explores the theoretical and practical implications of quantum decisionmaking for law. Part IV.A argues that, by identifying a set of principles that account for "rational" decisions and various heuristics and biases, quantum decisionmaking offers a useful theoretical framework for BLE. Part IV.B discusses a sample of eight law-related empirical predictions that arise from quantum decisionmaking and explores some implications of those predictions. Part V discusses caveats and current limitations of quantum decisionmaking, and Part VI provides concluding remarks.

II. RATIONAL CHOICE AND ITS LIMITS

In the 1950s and 1960s, courts and legal scholars began to embrace the idea that legal decisions could—and should—be informed by an analysis of their broader consequences.[38] But, to analyze these consequences, they needed some way to forecast how changes in legal incentives would affect citizens' decisions.[39] They needed a theory of decisionmaking.

36. In recent years, the thoughtful design of choice architecture—the contexts in which people make decisions—has been front and center in discussions of BLE, as carefully designed choice architecture can effectively shape citizens' decisions in desired ways without need for mandates. *See* Richard H. Thaler, Cass R. Sunstein & John P. Balz, *Choice Architecture*, *in* THE BEHAVIORAL FOUNDATIONS OF PUBLIC POLICY 428-439 (Eldar Shafir ed., 2013).

37. For readers interested in the "nuts-and-bolts" of how quantum models of judgments are constructed, we describe the construction of our example model in the Technical Appendix.

38. Ronald H. Coase, *The Problem of Social Cost*, 3 J. L. & ECON. 1 (1960); Guido Calabresi, *Some Thoughts on Risk Distribution and the Law of Torts*, 70 YALE L.J. 499 (1961); Cooter & Rubinfeld, *supra* note 4, at 1068.

39. Korobkin & Ulen, *supra* note 4, at 1055; *see also* Cooter & Rubinfeld, *supra* note 4, at 1068.

Around the same time, economist Gary Becker was embarking on a career-long effort to push the boundaries of economics beyond the traditional province of market activity.[40] Becker believed that a set of standard economic principles, referred to as rational choice theory, could be applied not only to markets but to all domains of human behavior.[41] Rational choice theory is typically operationalized using expected utility models of decisionmaking:[42] each of a decision maker's alternatives is assigned an expected utility value equal to a probability-weighted average of the utilities of its possible outcomes. The decision maker is assumed to gather an optimal amount of information to assess outcomes, calculate outcome probabilities according to classical (Bayesian) probability theory, and choose the alternative with the greatest expected utility.[43]

The law and economics movement took off when courts and scholars in need of a theory of human decisionmaking began importing rational choice theory, which Becker and his colleagues were already exporting.[44] During the 1970s and 1980s, the law and economics movement boomed, spreading its influence to every corner of the law.[45] As color-

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40. *See* GARY S. BECKER, THE ECONOMICS OF DISCRIMINATION (2d ed. 1971); RICHARD A. POSNER, THE ECONOMICS OF JUSTICE 3-4 (1983); Gary S. Becker, *Crime and Punishment: An Economic Approach*, 76 J. POL. ECON. 169 (1968); Samuel Issacharoff, *Can There Be a Behavioral Law and Economics?*, 51 VAND. L. REV. 1729, 1729 (1998) ("[T]he current sweep of law and economics would have been inconceivable without Gary Becker's insight into the application of neoclassical comparisons of marginal utility to the stuff of everyday life."); BECKER, *supra* note 7.

41. Per Becker, "all human behavior can be viewed as involving participants who [1] maximize their utility [2] from a stable set of preferences and [3] accumulate an optimal amount of information and other inputs in a variety of markets." BECKER, *supra* note 7, at 14.

42. *See* Jerome R. Busemeyer, *Cognitive Science Contributions to Decision Science*, 135 COGNITION 43, 43 (2015) ("The way to guarantee obedience to [the axioms of rational choice theory] is by using the [expected utility] formula to make choices . . . . The [expected utility] formula assigns a utility to each action, by computing a probability-weighted average of the utilities of outcomes produced by an action."); Korobkin & Ulen, *supra* note 4, at 1062-64 (describing the translation of rational choice theory into expected utility models, which are described as the "most dominant" conception of rational choice theory in modern microeconomics).

43. *See supra* notes 9-11.

44. Korobkin & Ulen, *supra* note 4, at 1055 ("[E]arly law-and-economics scholars imported from economics a series of assumptions about how people respond to incentives, known generally as 'rational choice theory.' ").

45. *See* Louis Kaplow & Steven Shavell, *Economic Analysis of Law*, *in* 3 HANDBOOK OF PUBLIC ECONOMICS 1666 (Alan J. Auerbach & Martin Feldstein eds., 2002) (surveying areas of economic influence on law); Richard A. Posner, *Some Uses and Abuses of Economics in Law*, 46 U. CHI. L. REV. 281 (1979) (same); Posner, *supra* note 4, at 1 ("Today there is an economic theory of property rights, of corporate and other organizations, of government and politics, of education, of the family, of crime and punishment, of anthropology, of history, of information, of racial and sexual discrimination, of privacy, even of the behavior of animals—and, overlapping all these but the last, of law.").

fully summarized by Professors Robert D. Cooter and Daniel L. Rubinfeld, "[l]ike the rabbit in Australia, the economic analysis of law found a vacant niche in the intellectual ecology, and filled it rapidly."[46]

The economic analysis of law has had tremendous influence on legal thought, but criticism of the approach—and its rationality assumptions in particular—hit critical mass in the late 1990s and early 2000s.[47] By that time, a body of research had been amassing for nearly two decades that posed substantial challenges for rational choice models of decisionmaking.[48] Building from Daniel Kahneman and Amos Tversky's seminal work on heuristics and biases,[49] this research by psychologists and behavioral economists demonstrated that, in a multitude of contexts,[50] people's judgments and decisions systematically violate the foundational assumptions of rational choice theory.[51] Chal-

46. Cooter & Rubinfeld, *supra* note 4, at 1068. Judge Richard A. Posner, in particular, was prolific during this law-and-economics explosion, collaborating with various co-authors to produce a sweeping series of articles applying economic principles to discrete nooks and crannies of law. *See, e.g.*, Richard A. Posner, *An Economic Approach to Legal Procedure and Judicial Administration*, 2 J. LEGAL STUDIES 399 (1973); Richard A. Posner & Isaac Ehrlich, *An Economic Analysis of Legal Rulemaking*, 3 J. LEGAL STUDIES 257 (1974); Richard A. Posner & Andrew M. Rosenfeld, *Impossibility and Related Doctrines in Contract Law: An Economic Analysis*, 6 J. LEGAL STUDIES 83 (1977); Richard A. Posner & William M. Landes, *Joint and Multiple Tortfeasors: An Economic Analysis*, 9 J. LEGAL STUDIES 517 (1980); Richard A. Posner & William M. Landes, *An Economic Theory of Intentional Torts*, 1 INT'L REV. OF L. & ECON. 127 (1981); Richard A. Posner & William M. Landes, *Causation in Tort Law: An Economic Approach*, 12 J. LEGAL STUDIES 109 (1983); Richard A. Posner, *Some Economics of Labor Law*, 51 U. CHI. L. REV. 988 (1984); Richard A. Posner, *An Economic Theory of Criminal Law*, 85 COLUM. L. REV. 1193 (1985); Richard A. Posner & Michael W. McConnell, *An Economic Approach to Issues of Religious Freedom*, 56 U. CHI. L. REV. 1 (1989); Richard A. Posner & William M. Landes, *An Economic Analysis of Copyright Law*, 18 J. LEGAL STUDIES 325 (1989).

47. *See supra* notes 12-14.

48. Korobkin & Ulen, *supra* note 4, at 1055-56 (noting that there was "simply too much credible experimental evidence that individuals frequently act in ways that are incompatible with the assumptions of rational choice theory" to be ignored any longer).

49. *See, e.g.*, Kahneman & Tversky, *Psychology of Prediction*, *supra* note 13; Tversky & Kahneman, *Judgment Under Uncertainty*, supra note 12. It should be noted, however, that the idea that rationality is bounded can be traced back to the earlier work of Herbert Simon. *See* Simon, *supra* note 15.

50. Indeed, the list of cognitive heuristics and biases has been characterized as "seemingly endless." Rachlinksi, *supra* note 3, at 748.

51. *See, e.g.*, RICHARD H. THALER, QUASI-RATIONAL ECONOMICS xxi (1994) ("In some well-defined situations, people make decisions that are systematically and substantively different from those predicted by the standard economic model."); Kathryn Zeiler, *Mistaken About Mistakes*, 2018 EUR. J. L. & ECON. 1, 1 (2018) (noting that the field of behavioral law and economics is "[m]otivated in large part by observed behavior that does not comport with predictions derived from neoclassical economic models of individual decision-making.").

lenges to the reliability of rational choice models often came with challenges to policies those models had been used to justify:[52] critics were concerned that "[w]here the economics is wrong, the law may be too."[53]

This concern spawned BLE. Speaking generally, BLE scholars endorse the concept of analyzing laws based on their predicted consequences, but urge that analysts should, in some contexts, eschew rational choice models and instead deploy situation-specific, empirically-grounded models that better reflect *actual* human behavior.[54] Further, BLE scholars urge that empirically-grounded models of human behavior are helpful not only for improving decisions about whether and how to implement substantive legal policies, but also for improving legal processes—for instance, to account for the potentially-error-causing heuristics and biases used by judges and juries in the process of deciding disputes.[55] Consider, as an example, Federal Rule of Evidence 404, which generally prohibits the use of "character evidence" to prove that a person acted in a particular manner on a particular occasion.[56] We spend some time with this example here because we will return to it later in the Article.

Imagine that the defendant in a murder case was previously convicted of the felony of armed robbery.[57] Under Rule 404, prosecutors generally cannot introduce evidence of the armed robbery conviction.[58] Viewed from a rational choice perspective, this prohibition tends to frustrate justice: evidence of the prior conviction is relevant, as "armed robbers are more likely than the average person to commit a murder."[59] When presented with evidence about the defendant's prior conviction, "[t]he rational juror should consider the base rate of murderers in the population (quite small) and update that probability" in accordance

52. Owen D. Jones, *Time-Shifted Rationality and the Law of Law's Leverage: Behavioral Economics Meets Behavioral Biology*, 95 NW. U. L. REV. 1141, 1141-43 (2001); *see also* Jolls, Sunstein & Thaler, *supra* note 3, at 1474 (suggesting that more accurate models of human behavior will yield "more accurate predictions and prescriptions about law").

53. Jones, supra note 54, at 1141.

54. *See* Jolls, Sunstein & Thaler, *supra* note 3, at 1473-74 (expressing the intent to "propos[e] a systematic framework for a behavioral approach to economic analysis of law, and us[e] behavioral insights to develop specific models and approaches addressing topics of abiding interest in law and economics")

55. Rachlinksi, *supra* note 18, at 1680-81 (noting that one of the principal insights of behavioral law and economics is that "[d]ecision making by judges and juries is frequently inaccurate in ways that can distort the civil and criminal justice systems" and that "[p]rocedural rules governing dispute resolution are, or should be, designed to prevent systematic errors in judgment from determining the outcome of adjudication.").

56. FED. R. EVID. 404(a). The prohibition is subject to certain exceptions.

57. Korobkin & Ulen, *supra* note 4, at 1087.

58. CHRISTOPHER B. MULLER & LAIRD. C. KIRKPATRICK, FEDERAL EVIDENCE 536-39 (2d ed. 1994).

59. Korobkin & Ulen, *supra* note 4, at 1087.

with classical probability theory (i.e., Bayes' Rule).[60] Excluding the evidence actually keeps jurors from being able to do this, thus *reducing* the likelihood that a rational juror will reach the correct verdict.[61]

But while prohibiting character evidence makes little sense when viewed through the lens of rational choice theory, it makes good sense from a BLE perspective.[62] A substantial body of empirical research suggests that decision makers often apply a "representativeness heuristic."[63] This heuristic posits that decision makers often estimate the likelihood of an event by comparing what they know about it to a set of prototypes already in their mind, and in doing so, may neglect or ignore base rates.[64] In the context of a murder trial, jurors given specific character evidence about the defendant are likely to "base their conclusion about the defendant's guilt on whether or not specific features about him look like stereotypical features of a murderer."[65] Thus, jurors are likely to *overweigh* evidence of a prior armed robbery conviction, violating classical probability theory and, thus, rational choice theory.[66] From a BLE perspective, then, it makes sense to exclude evidence of the prior conviction because it would predictably lead jurors to fallacious reasoning.

This is one of many examples across the law in which the law and economics approach and the BLE approach yield opposing policy prescriptions.[67] The reactions of the law and economics camp to BLE's prescriptions have been mixed. Some have expressed openness to BLE,

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60. *Id.*

61. *Id.*

62. *See id.* ("The law correctly excludes character evidence from consideration, however, if jurors are likely to ignore the base rate—that is, to ignore the fact that most [felons] are not murderers—and base their conclusion about the defendant's guilt on whether or not specific features about him look like stereotypical features of a murderer.").

63. *See, e.g.*, Kahneman & Tversky, *Psychology of Prediction*, *supra* note 13, at 241-43.

64. In perhaps the most famous illustration of this tendency, Kahneman and Tversky had participants read personality descriptions and estimate the probability that the person described was an engineer. Participants were told that the personality descriptions were selected at random from a group of 100, each of which described either an engineer or a lawyer. In one experimental condition, the participants were told that the underlying group consisted of 70 engineers and 30 lawyers; in the other experimental condition, the participants were told it consisted of 30 engineers and 70 lawyers. Tversky and Kahneman found that the descriptions of the individual (and, specifically, how well those descriptions fit the stereotype of either an engineer or a lawyer) had disproportionate weight on participants' responses, with participants largely ignoring the base rate information they had been given. *Id.*

65. Korobkin & Ulen, *supra* note 4, at 1087.

66. *Id.* ("[M]any jurors are likely to conclude that because the defendant has the appearance of a criminal (in that he has a felony conviction), he therefore must have committed the crime for which he is charged.").

67. For a sampling of divergent prescriptions, see Jolls, Sunstein & Thaler, *supra* note 3, at 1489-1541.

with a dose of skepticism about how useful it can be in practice.[68] Others have been quick to defend law and economics' reliance on rational choice theory, pointing out that, what it may lack in realism, it has more than made up for in utility over the years.[69] Some can be (and have been) characterized as "either uninterested or disdainful."[70] And, finally, some have turned the tables on BLE, pointing out some of the potential problems with the movement.[71]

This final group has focused its critique largely on BLE's atheoretical nature,[72] paralleling critiques in psychology journals of the heuristics and biases research on which BLE is built.[73] According to these critics, even if there are certain circumstances in which people's decisions deviate from classical rationality, BLE has not produced any common model or coherent framework that can account for or predict these deviations.[74] BLE scholarship has itself fed this critique at times,

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68. For discussion, see Issacharoff, *supra* note 40, at 1733-34; Rachlinski, *supra* note 5 (critiquing Hillman, *supra* note 4).

69. *See* Douglas G. Baird, *Introduction to Symposium: The Future of Law and Economics: Looking Forward*, 64 U. CHI. L. REV. 1129, 1131 (1997) ("The use of assumptions in economics is perhaps the aspect of the field that lawyers understand the least. Economists aim to capture as much of the dynamics of behavior as they can with the fewest possible assumptions. The question is not whether economists' assumptions are unrealistic, but whether they capture enough of what is at work to allow us to see basic forces operating in an otherwise impenetrable maze."); Robert E. Scott, *The Limits of Behavioral Theories of Law and Social Norms*, 86 VA. L. REV. 1603, 1645 (2000) ("It is perhaps understandable that legal analysts should try to mix economics and behavioral sciences to produce stronger legal arguments. But, in fact, a wrench grafted onto a hammer is not a better wrench or a better hammer, but simply an unwieldy tool."). For further discussion, see Thomas S. Ulen, *Rational Choice and the Economic Analysis of Law*, 19 J.L. & SOC. INQUIRY 487, 487-91 (1994); Jones, *supra* note 52, at 1142-43, 1146.

70. Thomas S. Ulen, *Growing Pains of Behavioral Law and Economics*, 51 VAND. L. REV. 1747, 1748 (1998).

71. *See* Arlen, *supra* note 11, at 1777 (critiquing behavioral law and economics for failing to provide a coherent alternative model of human behavior); Kelman, *supra* note 17, at 1579 (contending that behavioral law and economics replicates major flaws of the law and economics movement); Posner, *supra* note 18, at 1552 (arguing that behavioral law and economics is defined negatively as "economics minus the assumption that people are rational maximizers of their satisfactions."). For a recent and thorough analysis of these and other potential limitations of behavioral law and economics, see Jones, *supra* note 3, at 476.

72. *See* Arlen, *supra* note 9, at 1777; Jones, *supra* note 3; Posner, *supra* note 16, at 1552 (characterizing behavioral law and economics as "ad hoc" and "antitheoretical").

73. *See, e.g.*, Gerd Gigerenzer, *How to Make Cognitive Illusions Disappear: Beyond "Heuristics and Biases"*, *in* 2 EUROPEAN REVIEW OF SOCIAL PSYCHOLOLOGY 83, 101 (W. Stroebe & M. Hewstone eds., 1991) ("The absence of a general theory [explaining heuristics and biases] or even of specific models of underlying cognitive processes has been repeatedly criticized . . . but to no avail."); Anuj K. Shah & Daniel M. Oppenheimer, *Heuristics Made Easy: An Effort Reduction Framework*, 134 PSYCHOL. BULL. 207, 209 (2008) (describing a field with "a substantial list of heuristics but little in the way of comprehensive theory").

74. *See* Arlen, *supra* note 9, at 1768 ("Behavioral analysis of law does not have a coherent model of human behavior in part because . . . [b]ehavioral economists and cognitive psychologists generally have focused on demonstrating that people do not necessarily exhibit rational choice."); Jones, *supra* note 3, at 490 (contending that heuristics and biases seem to

"present[ing] the field as if it had little or no order, logic, underlying theory, or limiting principles"[75] or as a "haphazard collection of seemingly unrelated cognitive quirks."[76] Critics are understandably reluctant to abandon generally-applicable rational-choice models (which have proven valuable by any measure) for a collection of ad hoc models that may or may not apply in any specific situation.[77] After all, "[a] methodology that creates hidden psychological trump cards that scholars can play to contradict any assertion about human behavior cannot satisfy any legal scholar."[78]

To summarize, the law and economics camp and BLE camp agree that the analysis of law should be informed by models of human decisionmaking but disagree as to which models. Advocates of law and economics tend to favor rational choice models,[79] while advocates of BLE tend to prefer a collection of more context-specific, behavior-driven models.[80] The former approach is parsimonious and can be applied to generate predictions in a wide variety of situations; the latter approach more accurately predicts people's decisions (specifically, the "irrational" ones).[81] Quantum decisionmaking provides a middle ground: like rational choice, it offers a generally-applicable set of decision-making axioms, capable of generating testable empirical predictions; like BLE, it generates models that account for the "irrational" decisions people often make.[82]

Before moving on, we must address one specific domain of legal scholarship in which models of judgment and decisionmaking have

have been grouped together "not because they are thought to be tightly linked to one another," but because they all happen to violate rational choice models—"they form a category defined more by what they are not than by what they are."); Jones, *supra* note 52, at 1158 (noting that behavioral law and economics "is at base a movement founded on scattered discovered anomalies that, no matter how robust, are as yet wholly unconnected by theoretical foundation or adequate explanatory support").

75. Rachlinski, *supra* note 3, at 750.

76. Jon D. Hanson & Douglas A. Kysar, *Taking Behavioralism Seriously: The Problem of Market Manipulation*, 74 N.Y.U. L. REV. 630, 715 (1999).

77. *See, e.g.*, Arlen, *supra* note 9, at 1777; Jones, *supra* note 52, at 1158.

78. Rachlinksi, *supra* note 3, at 749.

79. *See, e.g.*, Kelman, *supra* note 17; Posner, *supra* note 16.

80. *See, e.g.*, Jolls, Sunstein & Thaler, *supra* note 3.

81. *See* Bruza, Wang, & Busemeyer, *supra* note 10, at 383 (noting that the "rational approach," which is favored in law and economics, "can be viewed as a 'top-down' deductive process, wherein the same basic axioms can be used to derive inferences and utilities across all environmental conditions," whereas the "heuristic approach," which is favored in BLE, "can be viewed as a 'bottom-up' inductive process in the sense that humans learn simple *ad hoc* rules that can be effective or not depending on the environmental conditions.").

82. *See id.* ("In common with the heuristic approach, [quantum decisionmaking] assumes that the human decision maker is subject to bounded rationality," but, "like the rational approach, inferences used for decisions are derived from basic axioms that derive a probability theory.").

been especially important: scholarship exploring how judges and jurors construe evidence. In this domain, "constructivist" models,[83] such as the "story model,"[84] "motivated reasoning,"[85] and "coherence-based reasoning,"[86] have provided valuable perspectives on how judges and jurors process evidence to reach factual and legal conclusions.[87] The insight common to these constructivist models is that decision makers do not weigh pieces of evidence separately and independently—an insight echoed in quantum decisionmaking.[88] But the focus of these constructivist models differs from that of quantum decisionmaking because the "dynamics [of the constructivist models] do not feature reasoning defects that defeat Bayesian information processing" but,

83. For a recent review of some of these models, see Dan M. Kahan, *Laws of Cognition and the Cognition of Law*, 135 COGNITION 56 (2015).

84. The "story model" posits that decision makers mentally organize the facts of a case through the use of story schema (familiar narratives, e.g., quiet loner lashes out violently), choosing the story that best fits the evidence presented according to certain criteria. *See* Nancy Pennington & Reid Hastie, *A Cognitive Theory of Juror Decision Making: The Story Model*, 13 CARDOZO L. REV. 519, 526-527 (1991); Nancy Pennington & Reid Hastie, *Explaining the Evidence: Tests of the Story Model for Juror Decision Making*, 62 J. PERSONALITY & SOC. PSYCH. 189, 191-92 (1992). The story model draws heavily from research on discourse comprehension, with the key idea being that jurors are engaged in a *constructive* process in which incoming information is organized into mental representations. *See, e.g.*, Tom Trabasso & Paul Van Den Broek, *Causal Thinking and the Representation of Narrative Events*, 24 J. MEMORY & LANGUAGE 612, 612 (1985).

85. "Motivated reasoning," assumes that decision makers unconsciously conform their evaluation of evidence to match goals other than factual accuracy (e.g., punishing "disgusting" behavior). *See* Janice Nadler & Mary-Hunter McDonnell, *Moral Character, Motive, and the Psychology of Blame*, 97 CORNELL L. REV. 255 (2011); Avani Mehta Sood, *Motivated Cognition in Legal Decision Making-An Analytic Review*, 9 ANN. REV. LAW & SOC. SCI. 307 (2013); Avani Mehta Sood & John Darley, *Plasticity of Harm in the Service of Criminalization Goals*, 100 CAL. L. REV. 1313 (2012).

86. "Coherence-based reasoning" supposes that evaluating evidence can be a *recursive* process: because decision makers are motivated to avoid complex, challenging decisions, they have a tendency to reconfigure their mental models of evidence until choices are easy (i.e., until evidence in support of the chosen verdict is strong and evidence in support of alternatives is weak). *See* Simon, *supra* note 3; *see also* Leon Festinger, A THEORY OF COGNITIVE DISSONANCE (1957); Fritz Heider, *Attitudes and Cognitive Organization*, 21 J. PSYCHOL. 107 (1946); Dan Simon & Keith J. Holyoak, *Bidirectional Reasoning in Decisionmaking by Constraint Satisfaction*, 128 J. EXP. PSYCHOL.: GEN. 3 (1999). The coherence-based reasoning view traces its roots to psychological findings suggesting that people prefer their actions to be consistent with their beliefs and attitudes (and may revise beliefs and attitudes based on their actions).

87. Kahan, *supra* note 83, at 56 (These models model the process by which "a decisionmaker (typically a juror) . . . find[s] facts and appl[ies] rules that specify the significance of such facts."). Note this is not intended be an exhaustive survey of constructivist models. These, and other, constructivist models are psychologically-based alternatives to the "dominant" view that people process evidence in a linear, Bayesian fashion. *See also* Bilz, *supra* note 14, at 429. ("The currently dominant view of how people process evidence and draw conclusions is linear and Bayesian.").

88. Trueblood & Busemeyer, *supra* note 33, at 1518-19 (providing a quantum account of the phenomenon that the order in which evidence is presented affects final judgments).

"[r]ather, they address how information can shape a variety [of] cognitive inputs that a Bayesian framework *presupposes*."[89] In other words, the aim of these constructivist models is generally not to explain the heuristics and biases that animate BLE.[90] The express aim of quantum decisionmaking, on the other hand, is to account for those heuristics and biases.[91]

III. INTRODUCING QUANTUM DECISIONMAKING

The argument that the catalog of heuristics and biases is haphazard and atheoretical has appeared in psychology literature as well as in law reviews.[92] Thus, it should come as no surprise that researchers in psychology departments have been working to address this critique. One promising theory that has emerged from this work is "quantum decisionmaking."[93]

Quantum decisionmaking resembles the heuristics and biases approach employed by BLE in that "it assumes that the human decision maker is subject to bounded rationality," but it resembles rational choice theory in that it assumes the "inferences used for decisions are derived from basic axioms that define a probability theory."[94] The defining feature of quantum decisionmaking is that it uses a *different* probability theory.[95] Specifically, quantum decisionmaking assumes that decision makers judge probabilities in accordance with *quantum*

89. Kahan, *supra* note 83, at 56 (emphasis in original). Indeed, these models have been referred to as "untamed Bayesianism." *Id.*

90. *Id.*; *see also* Simon, *supra* note 3, at 517 (noting that research on coherence-based reasoning differs from research on heuristics and biases, as the "two bodies of research examine different cognitive phenomena.").

91. Bruza, Wang & Busemeyer, *supra* note 8, at 383. It should be noted that quantum decisionmaking is not the first or only theoretical perspective on this puzzle. Other valuable perspectives include, *inter alia*, prospect theory, prospective reference theory, and the biologically-based theory of time-shifted rationality. *See* Daniel Kahneman & Amos Tversky, *Prospect Theory: An Analysis of Decision under Risk*, 47 ECONOMETRICA 263 (1979) (prospect theory); W. Kip Viscusi, *Prospective Reference Theory: Toward an Explanation of the Paradoxes*, 2 J. RISK & UNCERTAINTY 235 (1989); Jones, *supra* note 52.

92. *E.g.*, Busemeyer et al., *supra* note 26 at 193; Shah & Oppenheimer, *supra* note 73, at 207-09.

93. Work on quantum decisionmaking has appeared in top journals in the field of psychology, including *Proceedings of the National Academy of Sciences*, *Proceedings of the Royal Society*, *Psychological Review*, and *Brain and Behavioral Science*, among others. *See* Emmanuel M. Pothos, Jerome R. Busemeyer & Jennifer S. Trueblood, *A Quantum Geometric Model of Similarity*, 120 PSYCHOL. REV. 679 (2013); Pothos & Busemeyer, *supra* note 22; Wang et al., *supra* note 23; Yearsley & Pothos, *supra* note 35. Both *The Journal of Mathematical Psychology* (2009) and *Trends in Cognitive Sciences* (2013) have recently dedicated special issues to the quantum approach.

94. Bruza, Wang & Busemeyer, *supra* note 8, at 383.

95. For a discussion of different probability theories with potential relevance to decisionmaking, see Louis Narens, *Alternative Probability Theories for Cognitive Psychology*, 6 TOPICS IN COGNITIVE SCI. 114 (2014).

probability theory.[96] As explained below, this difference allows quantum decisionmaking to capture many of the heuristics and biases at the heart of BLE[97]—and to predict contexts in which we might observe new ones.[98]

This Part introduces the basic principles of quantum decisionmaking to a legal audience. Part III.A provides a conceptual introduction, summarizing the core tenets of quantum decisionmaking and highlighting key differences from rational choice theory. Part III.B then provides an example of a quantum model of a legal judgment and illustrates how it accounts for heuristics and biases. We note that the paper is structured such that readers who are less interested in the technical aspects of quantum models than in their legal implications can bypass Part III.B, proceeding from Part III.A to Part IV.

A. A Conceptual Introduction to Quantum Decisionmaking

Quantum decisionmaking is a mathematical theory of how people make judgments and decisions.[99] The central idea is that cognitive models based in quantum probability theory can successfully describe and predict judgments and decisions (legal and otherwise).[100] This may raise several questions for the reader: (1) what are cognitive models?, (2) how do cognitive models relate to probability theories?, (3) what is quantum probability theory?, and (4) why would one apply it to human decisionmaking? The rest of this section is intended to answer these questions and, in the process, unpack the key concepts related to quantum decisionmaking. Part IV then explores the implications of these concepts for law.

1. Cognitive Models

Cognitive models are approximations (typically mathematical) of how our minds work.[101] They are intended to be algorithms—rules that

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96. Bruza, Wang & Busemeyer, *supra* note 10, at 383-84.

97. *See infra* Part IV.A.

98. *See infra* Part IV.B.

99. Quantum decisionmaking is one part of a larger field of research called "quantum cognition," which also includes quantum-based models to describe other aspects of human cognition including memory and perception. Busemeyer & Wang, *supra* note 23, at 163 ("Quantum cognition is a new research program that uses mathematical principles from quantum theory as a framework to explain human cognition, including judgment and decisionmaking, concepts, reasoning, memory, and perception.").

100. *Id.*; *see also* James M. Yearsley & Jerome R. Busemeyer, *Quantum Cognition and Decision Theories: A Tutorial*, 74 J. MATHEMATICAL PSYCHOL. 99 (2016).

101. *See, e.g.*, JEROME R. BUSEMEYER & ADELE DIEDERICH, COGNITIVE MODELING 2-4 (2015) (describing the hallmarks of cognitive modeling). Behavioral law and economics scholars have been interested in modeling since the dawn of the discipline; *see* Cass R. Sunstein, *Behavioral Analysis of Law*, 64 U. CHI. L. REV. 1175, 1175 (1997) (noting that heuristics and biases are not unpredictable; they "can be described, used, and sometimes even modeled.").

help us make predictions about human cognition—and they usually focus on one or two specific mental processes.[102] Researchers use cognitive models to explain existing empirical observations, predict future observations, or, ideally, both.[103] Psychologists have developed cognitive models of all sorts of processes, including perception,[104] memory,[105] and, most relevant for our purposes, decisionmaking.[106]

For example, one could generate several simple (and exceedingly specific) cognitive models of how people decide on a restaurant for dinner. Model #1: People bring to mind the best meal they've experienced at each restaurant and mentally compare those meals to one another, opting for the restaurant that gave them the best meal in the past. Model #2: People have a sense, based on their experience, of what percentage of the time they get satisfactory meals at each restaurant, and opt for the restaurant that gives them the highest probability of a satisfactory meal. Model #3: People act as though they combine these two pieces of information into a "score" in some manner. For example, the score may be the deliciousness of the best meal people remember at each restaurant option (say a 10 at Restaurant A and a 5 at Restaurant B) multiplied by people's perceived probability of getting a satisfactory meal at that restaurant (assume 40% at Restaurant A and 90% at Restaurant B). People then opt for the restaurant option with the highest score (in our example, Restaurant B, because Restaurant B's score of 4.5 would be higher than Restaurant A's score of 4).

We can compare our three cognitive models to one another to see which one best "fits" the decisions that people actually make.[107] Researchers could, for example, ask people who have already made dinner choices questions related to the three models (e.g., by asking diners about the best meal and the proportion of satisfactory meals they have experienced at each of the restaurants they considered) and use the

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102. *E.g.*, BUSEMEYER & DIEDERICH, *supra* note 101, at 6 ("[A]ll models are deliberately constructed to be simple representations that only capture the essentials of the cognitive systems" that they model.).

103. *Id.* at 4-5.

104. *See, e.g.*, Vicki Bruce & Andy Young, *Understanding Face Recognition*, 77 BRITISH J. PSYCHOL. 305 (1986) (modeling face perception); Gerald Lee Lohse, *A Cognitive Model for Understanding Graphical Perception*, 8 J. OF HUMAN-COMPUTER INTERACTION 353 (1993) (modeling graphical perception); Theodore B. Jaeger, *Assimilation and Contrast in Geometrical Illusions: A Theoretical Analysis*, 8 PERCEPTUAL & MOTOR SKILLS (1999) (modeling perception of geometrical illusions).

105. For a review of cognitive models of working memory, see Mark D'Esposito, *From Cognitive to Neural Models of Working Memory*, 362 PHIL. TRANSACTIONS OF THE ROYAL SOC'Y OF LONDON 761 (2007).

106. *See. e.g.*, Mandeep K. Dhami, *Psychological Models of Professional Decision Making*, 14 PSYCHOL. SCI. 175 (2003) (comparing alternative models for judicial decisions).

107. For a discussion of fitting cognitive models, see BUSEMEYER & DIEDERICH, *supra* note 101, at 6-7 (describing the process of estimating parameters for competing cognitive models based on data, and then comparing the models on their ability to explain the data).

responses to try to determine which of the models best explains why people chose to eat where they did. Researchers could also ask people to report, on the front end, the model-relevant information and then see which model best predicts individuals' subsequent dinner choices.

Of course, the cognitive models that psychologists work with tend to concern a far broader set of decisions—with far more important consequences—than our dinner example. These models are highly relevant to heuristics and biases. One goal of cognitive modelers has been to generate coherent models of decisionmaking that both account for and predict the heuristics and biases demonstrated by Kahneman, Tverksy, and their colleagues.[108]

2. *The Relationship Between Cognitive Models and Probability Theories*

Many cognitive models of decisionmaking—and all of the models associated with traditional law and economics—are rational choice models.[109] These models posit that a decision maker will tend to choose the alternative that offers him or her the greatest *expected utility*.[110] Expected utility can be thought of as a score reflecting how attractive a particular decision alternative is. This "score" reflects an average of (1) the utility (i.e., satisfaction) the decision maker would derive from the various possible outcomes of choosing that alternative, weighted

108. *E.g.*, Bruza, Wang & Busemeyer, *supra* note 8, at 387 (noting that quantum decisionmaking has been inspired by the "steady accumulation of puzzling, even paradoxical, cognitive phenomena that violate the axioms upon which classical probability theory (and hence Bayesian inference) is based.").

109. *See supra* Part II.

110. *See* J. FRANK YATES, JUDGMENT AND DECISION-MAKING 241-303 (1990) (providing an accessible introductory discussion of expected utility and closely related theories of decisionmaking). Note that rational choice theory does not posit that people actually reason or deliberate in this manner. Rather, it posits that people make decisions as if they do so. Some rational choice theorists assume that the decision maker always chooses the alternative with the greatest expected utility. Others make probabilistic predictions about the decision maker's choice using "choice rules," such as Luce's choice rule. *See, e.g.*, R. DUNCAN LUCE, INDIVIDUAL CHOICE BEHAVIOR: A THEORETICAL ANALYSIS (1959) (introducing Luce's choice rule); Timothy J. Pleskac, *Decision and Choice: Luce's Choice Axiom*, *in* INTERNATIONAL ENCYCLOPEDIA OF SOCIAL & BEHAVIORAL SCIENCE 896 (J.D. Wright ed., 2015) (probabilistic choice rules are used to reflect that alternatives are selected only with some probability).

by (2) the probabilities (typically subjective)[111] of those outcomes occurring.[112]

For example, imagine that a plaintiff sues a defendant for both trademark infringement and defamation, seeking $100,000 of damages relating to each claim ($200,000 total). Assume, for the sake of simplicity, that there are no plausible disputes about damage amounts, such that each claim represents an all-or-nothing proposition for the plaintiff. Prior to trial, the defendant offers to settle the case for $50,000. The plaintiff has a decision to make. Her alternatives are to accept the settlement offer or take her chances at trial. The expected utility of the settlement option is $50,000:[113] $EU_{settle} = 100\% * \$50,000$. The expected utility of the trial option is equal to an average of the various possible outcomes (prevailing on neither claim = $0; prevailing on the trademark claim but not the defamation claim = $100,000; prevailing on the defamation claim but not the trademark claim = $100,000; prevailing on both claims = $200,000) weighted by the plaintiff's assessment of the probabilities of each of those outcomes occurring (we will call them w, x, y, and z,): $EU_{trial} = (w\% * 0) + (x\% * \$100,000) + (y\% * \$100,000) + (z\% * \$200,000)$. In this situation, the expected utility of the trial alternative—and therefore, the plaintiff's decision between settlement and trial—depends on the plaintiff's probability assessment.

Importantly—indeed, for our purposes, this is the key—rational choice theory assumes that the plaintiff's probability assessment is governed by the rules of *classical probability theory*.[114] Probability the-

111. These probabilities may be either the "objective probabilities" based on the state of the world—the traditional formulation of expected utility theory—or the "subjective probabilities" based on the decision maker's impressions of the objective probabilities. The latter view has been more popular in recent years and is captured by approaches to utility like Kahneman and Tversky's "prospect theory." Kahneman & Tversky, *supra* note 91 (introducing prospect theory). Indeed, some have argued that subjective probabilities are the only realistic options for capturing how people decide. *See* Bruno de Finetti, *Logical Foundations and Measurement of Subjective Probability*, 34 ACTA PSYCHOLOGICA 129, 129 (1970) (asserting that subjective probability is the "only meaningful interpretation" of probability since probability is a creature of the mind and does not exist in a substantial sense); Daniel Kahnemann & Amos Tversky, *Subjective Probability: A Judgment of Representativeness*, *in* THE CONCEPT OF PROBABILITY IN PSYCHOLOGICAL EXPERIMENTS 25 (1972) ("The decisions we make, the conclusions we reach, and the explanations we offer are usually based on our judgments of the likelihood of uncertain events such as success in a new job, the outcome of an election, or the state of the market."). For an introductory discussion of subjective probability and prospect theory, see YATES, *supra* note 110, at 282-303 (introducing variants of expected utility theory built on subjective impressions of probability).

112. *E.g.*, YATES, *supra* note 110.

113. We assume that utility is commensurate to dollars for the sake of example.

114. Bruza, Wang & Busemeyer, *supra* note 8, at 383, 385 (abstract) ("Although rational models of cognition have become prominent and have achieved much success, they adhere to the laws of classical probability theory . . . .").

ories provide formal methods for assigning likelihood values to uncertain events.[115] In classical probability theory, the entire universe of possible events form a "sample space," such that the probability of *something* from the sample space happening is 1, or 100%.[116] Subsets of the sample space represent particular outcomes or collections of outcomes, called "events."[117] In our example, the plaintiff must judge the probabilities of: (1) the event that she prevails on neither claim, (2) the event that she prevails on her trademark infringement claim but not her defamation claim, (3) the event that she prevails on her defamation claim but not her trademark infringement claim, and (4) the event that she prevails on both claims.

Classical probability theory posits that the plaintiff's judgments of probability can be represented in a classical sample space (often illustrated as a Venn diagram).[118] Figure 1 depicts one possible classical sample space. You can think of this sample space as representing the plaintiff's impressions of the case at any given moment in time. The plaintiff's perception of the relative probabilities of the events (i.e., the sizes of subspaces in the sample space) can be revised as the plaintiff

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115. *E.g.*, Yearsley & Busemeyer, *supra* note 100, at 100. Note that "probability theory does not determine the probabilities of uncertain events—it merely imposes constraints on the relations among them." Tversky & Kahneman, *The Conjunction Fallacy*, *supra* note 13, at 293-94. For an introduction to probability theory and basic terminology, see JESSICA HWANG & JOSEPH K. BLITZSTEIN, INTRODUCTION TO PROBABILITY (2014); BUSEMEYER & BRUZA, *supra* note 21; YATES, *supra* note 110, at 112-146.

116. *See, e.g.*, Bruza, Wang & Busemeyer, *supra* note 8, at 384-85; YATES, *supra* note 110, at 115.

117. Busemeyer & Trueblood, *supra* note 33, at 3; YATES, *supra* note 110, at 114. Note that some sources refer to "events" as "elements," *see, e.g.*, HWANG & BLITZSTEIN, *supra* note 115, but we use the term "events" throughout this Article.

118. *See, e.g.*, YATES, *supra* note 110, at 115 (illustrating sample space with Venn diagram).

gets more information (e.g., when the plaintiff reviews pre-trial discovery related to the defendant's case, learns the outcome of an important motion, or finds out that the defendant regularly golfs with the judge).

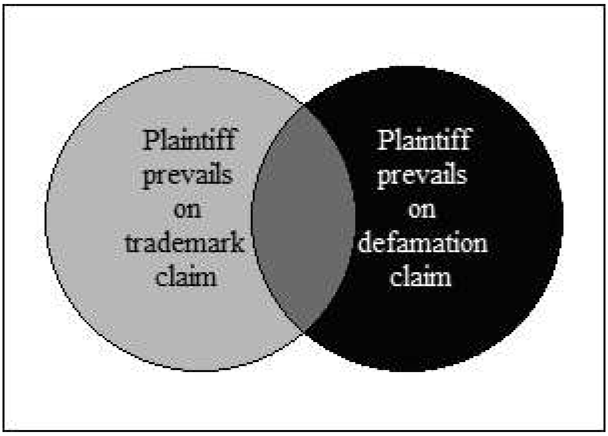

Figure 1. Sample space for the plaintiff's judgments. The square represents the sample space, the light gray circle represents the event "plaintiff prevails on trademark claim," and the black circle represents the event "plaintiff prevails on defamation claim." The overlap of the two circles (shaded dark gray) represents the joint event in which the plaintiff prevails on both the trademark claim and the defamation claim.

Under classical probability theory, certain rules constrain the probabilities that the plaintiff assigns to events.[119] These rules may sound familiar if the reader has taken statistics courses. A few examples follow.

- The probability that the plaintiff prevails on her trademark claim and the probability that the plaintiff does not prevail on her trademark claim sum to 1, the total area of the sample space. (This reflects that the plaintiff will either prevail on her trademark claim or not.)[120]
- The joint probability that the plaintiff both prevails on her trademark claim and prevails on her defamation claim (the intersection of the two circles in Figure 1 given by the dark gray region) must be (1) less than or equal to the standalone probability that the plaintiff prevails on her trademark claim (light

119. *Id.*
120. *Id.* at 119.

gray region), and (2) less than or equal to the standalone probability that the plaintiff prevails on her defamation claim (black region).[121]

- The conditional probability that the plaintiff prevails on her trademark claim, given that the plaintiff prevails on her defamation claim, equals the joint probability that the plaintiff both prevails on her trademark claim and prevails on her defamation claim (dark gray region), divided by the probability that the plaintiff prevails on her defamation claim (black region).[122]

Although rational choice models impose these rules on people's judgments of probabilities, research demonstrates that people's *actual judgments* often violate these rules. For example, consider the conjunction fallacy. The conjunction fallacy is illustrated by Tversky and Kahneman's "Linda problem,"[123] a common example in the legal scholarship concerning judgment and decisionmaking.[124] As presented by Tversky and Kahneman, the Linda problem reads as follows:

> Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.
>
> Which is more probable?
>
> 1. Linda is a bank teller.
>
> 2. Linda is a bank teller and is active in the feminist movement.[125]

People tend to rate the second statement as more probable than the first.[126] This response, however, violates classical probability theory, specifically, the rule following the second bullet point above.[127] Just as the joint probability that the plaintiff both prevails on her trademark claim and prevails on her defamation claim must be less than or equal

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121. Bruza, Wang & Busemeyer, *supra* note 8, at 364 (In classical theory, "the probability of an event *A* can never be less than the probability of the conjunction of *A* with another event *B*.").

122. YATES, *supra* note 110, at 129-30.

123. Tversky & Kahneman, *The Conjunction Fallacy*, *supra* note 13.

124. *E.g.*, Gregory Mitchell, *Taking Behavioralism Too Seriously? The Unwarranted Pessimism of the New Behavioral Analysis of Law*, 43 WM. & MARY L. REV. 1907, 1984-85, n. 157 (2001); Cass R. Sunstein, *Review: Hazardous Heuristics*, 70 U. CHI. L. REV. 751, 764-65 (2003); Andrew J. Wistrich, Jeffrey J. Rachlinski & Chris Guthrie, *Heart Versus Head: Do Judges Follow the Law or Follow Their Feelings?*, 93 TEX. L. REV. 855, 864-65 (2014); Rachlinski, *supra* note 18, at 1682-84.

125. Tversky & Kahneman, *The Conjunction Fallacy*, *supra* note 13, at 297.

126. *Id.* at 243.

127. *Id.*

to the standalone probability that the plaintiff prevails on her trademark claim, the joint probability of Linda being a bank teller *and* an active feminist must be less than or equal to the standalone probability of Linda being a bank teller.[128] After all, if "Linda is a bank teller and is active in the feminist movement,"[129] then she is necessarily "a bank teller."[130]

Importantly, the conjunction fallacy is not a cognitive quirk unique to undergraduate research participants. It also occurs in federal judges.[131] Further, the fallacy is not simply a product of a lack of motivation. Even people gambling with real dollars—who presumably get more utility from winning more money[132]—violate the conjunction fallacy and make "irrational" decisions when betting.[133]

The conjunction fallacy is only one of many documented heuristics and biases that illustrate an uncomfortable truth for rational choice theorists: people's probability judgments are sometimes *incoherent* from a classical probability perspective and, thus, from a rational choice perspective.[134] To be clear, this is different than just saying people sometimes make *bad* judgments. Rather, in many well-documented circumstances, people's judgments violate the foundational rules of classical probability theory.[135] Such "irrationalities" in proba-

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128. More generally, under classical probability theory, the probability of any given event X occurring cannot be less than the probability of that event X *and* another event Y occurring (or, stated positively, it must be true that $p(X) \geq p(X\&Y)$). *See* YATES, *supra* note 110, at 125.

129. Tversky & Kahneman, *The Conjunction Fallacy*, *supra* note 13, at 297.

130. *Id.*

131. Chris Guthrie, Jeffrey J. Rachlinski & Andrew J. Wistrich, *The "Hidden Judiciary": An Empirical Examination of Executive Branch Justice*, 58 DUKE L.J. 1477, 1510-12 (2009).

132. Psychologists and behavioral economists studying heuristics and biases have traditionally been especially interested in gambles, since they provide a context in which the utility of different outcomes is already quantified (the preference for winning money over losing money tends to be universal).

133. This has been demonstrated a number of times, including in the following articles: Maya Bar-Hillel & Efrat Neter, *How Alike it is Versus How Likely it is: A Disjunction Fallacy in Probability Judgments*, 65 J. PERSONALITY & SOC. PSYCHOL. 1119 (1993) Ashley Sides et al., *On the Reality of the Conjunction Fallacy*, 2 MEMORY & COGNITION 191 (2002); Tversky & Kahneman, *The Conjunction Fallacy*, *supra* note 15, at 303 (in one experiment involving a dice-rolling game, participants bet irrationally whether actually playing for money or responding to hypotheticals).

134. YATES, *supra* note 110, at 118-140, 249-253, 277, 296-303 (discussing examples).

135. *See id.* (discussing examples of incoherence); George Wright & Peter Whalley, *The Supra-additivity of Subjective Probability*, *in* FOUNDATIONS OF UTILITY AND RISK THEORY WITH APPLICATIONS, 233-244 (1983) (demonstrating that people's subjective likelihood estimates for a list of exclusive events consistently sums up to more than 100%); Tversky & Kahneman, *The Conjunction Fallacy*, *supra* note 13, at 294 (noting that the rule that a conjunction cannot be more probable than one of its constituents "is valid for any probability assignment on the same sample space.").

bility judgments result in "persistent and systematic discrepancies between how" rational choice theory predicts people will decide and how people actually decide.[136]

But are there adjustments we can make to our models of decisionmaking to capture or correct for incoherent probability judgments? More than you might expect. We can try to account for them by adding features to rational choice models, at the cost of parsimony.[137] Or, we can try to account for them by using a different probability theory.[138] Indeed, while most people have only been exposed to one probability theory (the classical probability theory discussed above), it turns out that there are actually multiple *other* viable probability theories, each with their own axioms that can be used in building models of decisionmaking.[139] There is no particularly deep reason to assume that humans make probability judgments in accordance with classical probability theory rather than one of the alternative theories. Psychologists have recently become especially interested in one alternative: "quantum probability theory."[140]

3. *Applying Quantum Probability Theory to Human Decisionmaking*

Introducing quantum probability theory in the context of human decisionmaking may seem puzzling. Quantum probability theory was developed within the field of quantum physics, and it governs the behavior of subatomic particles.[141] Why, then, would we draw from quantum probability theory to model human decisionmaking? While it may initially seem strange, it is not intrinsically any stranger than applying classical probability theory to human decisionmaking, as rational choice theory does.[142] Classical probability theory also developed in the

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136. YATES, *supra* note 1110, at 277; *see also id.* at 296-303 (discussing evidence for and against prospect theory, a cousin of classical expected utility theory).

137. *See, e.g.*, Fintan Costello & Paul Watts, *Surprisingly Rational: Probability Theory Plus Noise Explains Biases in Judgment*, 121 PSYCHOL. REV. 463 (2014).

138. Narens, *supra* note 95, at 114 (commenting on alternative probability theories with potential use in cognitive modeling).

139. *Id.*; Bruza, Wang & Busemeyer, *supra* note 8, at 383 (citing LOUIS NARENS, PROBABLISTIC LATTICES: WITH APPLICATIONS TO PSYCHOLOGY (2015)).

140. Bruza, Wang & Busemeyer, *supra* note 10, at 383.

141. *Id.*

142. *See* Pam Frost Gorder, *You're Not Irrational, You're Just Quantum Probabilistic*, OHIO ST. NEWS (Sept. 14, 2015), https://phys.org/news/2015-09-youre-irrational-quantum-probabilistic-human.html [https://perma.cc/Y9KC-YQZU] (quoting Professor Zheng Wang as follows: "For example, we ask, what is the probability that a person will act a certain way or make a certain decision? Traditionally, those models are all based on classical probability theory—which arose from the classical physics of Newtonian systems. So it's really not so exotic for social scientists to think about quantum systems and their mathematical principles, too.") [hereinafter Gorder]. Calls to incorporate insights from quantum theory into decisionmaking science began as far back as the 1920's. *See* WENDT, *supra* note 25, at 4 ("This

field of physics.[143] Further, there is a compelling analogy between particle physics and human judgment that suggests quantum probability theory may be a good fit:

> Judgment is not a simple readout from a pre-existing or recorded state; instead, it is constructed from the question and the cognitive state created by the current context. From this first point, it then follows that (b) drawing a conclusion from one judgment changes the context, which disturbs the state of the cognitive system, and the second point implies (c) changes in context and state produced by the first judgment affects the next judgment, producing order effects, so that (d) human judgments do not obey the commutative rule of Boolean logic. (e) Finally, these violations of the commutative rule lead to various types of judgment errors according to classic probability theory. If we replace "human judgment" with "physical measurement" and replace "cognitive system" with "physical system," then these are the same points faced by physicists in the 1920s that forced them to develop quantum theory. In other words, quantum theory was initially invented to explain noncommutative findings in physics that seemed paradoxical from a classical point of view. Similarly, noncommutative findings in cognitive psychology, such as order effects on human judgments, suggest that classical probability theory is too limited to provide a full explanation of all aspects of human cognition.[144]

The key insight to draw from this analogy is that—for particles and judgments alike—*context matters*. The next judgment you make may be influenced by the last one, which may have been influenced by the one before that.[145] It is not easy to account for these contextual influences on judgments in a classical probability framework.[146] A quantum probability framework, however, can capture these influences easily—

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is not the first call for a quantum social science. Already in 1927—just weeks after the Solvay conference marking the culmination of the quantum revolution—the President of the American Political Science Association, William Bennett Munro, challenged social scientists to come to grip with the new physics."); Munro, *supra* note 27, at 2 ("New truths cannot be quarantined. No branch of knowledge advances by itself. In its progress it draws others along. By no jugglery of words can we keep Mind and Matter and Motion in watertight compartments . . . ."). Interestingly, foundational concepts in the quantum physics revolution may have been imported from psychology. *See* Zheng Wang et al., *The Potential of Using Quantum Theory to Build Models of Cognition*, 5 TOPICS IN COGNITIVE SCI. 672, 678 (2013) ("It is another interesting twist in the history of science that [quantum physicist] Bohr actually imported [the concept of complementarity to physics] from psychology, where it had been coined by William James.").

143 Gorder, *supra* note 144.

144. Busemeyer et al., *supra* note 26.

145. Busemeyer & Trueblood, *supra* note 33, at 2.

146. Bruza, Wang & Busemeyer, *supra* note 10, at 384 (noting that the logic underlying classical probability theory implies that combinations of events are always commutative).

indeed, quantum probability theory was specifically developed to do so.[147]

The assumptions of quantum probability theory differ from those of classical probability theory on three critical, related points: definiteness, sample spaces, and commutativity. We discuss each in turn.

Definiteness. In classical probability theory, judgments arise from *definite states.*[148] In other words, a decision maker always has a defined mental representation of the relevant probabilities in his or her mind. In the context of our settlement example, we would say that, at any stage of the litigation, the plaintiff has defined opinions of how likely she is to prevail on her trademark claim and on her defamation claim. That opinion may change repeatedly, in light of additional information, but at any given moment the plaintiff has definite opinions. When a measurement occurs (i.e., when the plaintiff makes her settlement decision), she simply "reads out" the probabilistic judgments that already exist in her mind at that moment and uses them to guide her decisionmaking.[149]

In the quantum view, on the other hand, judgments arise from *indefinite states*.[150] People can remain in a state of "I don't know" about uncertain outcomes until they are prompted to make a firm judgment.[151] When a firm judgment is prompted, it is *constructed* based on the interaction of the indefinite state in the decision maker's mind and the question that is posed. Multiple judgments are possible until the moment of measurement (i.e., decision), but any given measurement produces only one.[152] This indefiniteness reflects our psychological experience in everyday decisionmaking. Our attitudes are not always

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147. *See id.* at 384 (observing that the "mathematical structure of quantum theory might lend itself to the modeling of human cognition because many cognitive phenomena crucially depend on the sequence or order of the cognitive processes and measurements (e.g., judgments and decisions), and quantum theory was initially developed to address order effects of measurements in physics.").

148. Busemeyer & Trueblood, *supra* note 33, at 1-2.

149. *See* Trueblood & Busemeyer, *supra* note 30, at 1 (in classical theory, "[t]he process of imposing [a] measurement has no effect on [one's cognitive] state other than to simply record it.").

150. *See* Bruza, Wang & Busemeyer, *supra* note 8, at 384 ("[Q]uantum probability theory assumes that, at any moment, a system is in an indefinite (technically dispersed) superposition state until a measurement is performed on the system. To be in a superposed state means that all possible definite states have the potential for being actualized, but only one of them will become actual upon measurement.").

151. This concept—called superposition in physics—"resonates with the fuzzy, ambiguous, uncertain feelings in many psychological phenomena." *Id.; see also* Busemeyer & Trueblood, *supra* note 33, at 2 (noting that an indefinite state "provides a better representation of the *conflict, ambiguity, or uncertainty* that people experience at each moment.").

152. Quantum probability theory allows a decision maker to be in an indefinite state until a measurement occurs (i.e., the decision maker has to make a decision). Yearsley & Pothos, *supra* note 35, at 6. Then, when the measurement (decision) occurs, the indefinite state "collapse[s]" to a state consistent with the measured outcome (the choice). *Id.*

clear—we often feel conflict, ambiguity, or uncertainty. Yet, when we are asked questions (ranging from "what do you want for dinner?"[153] to "do you want to accept the settlement offer?") we typically take a moment to think about it and then provide a clear answer.[154] Indefiniteness in judgments is analogous to the famous "Schrödinger's cat" thought experiment in physics: a cat inside a box might be either dead or alive (there is some probability of either) until we open the box and check, at which point the cat must be either dead or alive.[155] From a quantum perspective, "it's as if each decision we make is our own unique Schrödinger's cat."[156]

Sample spaces. Classical probability models typically assume a single sample space.[157] In other words, they assume that decision makers represent and consider all possible events together, simultaneously, in an exhaustive, universal sample space (i.e., one large mental Venn diagram).[158] In our example, the plaintiff's impressions concerning the likelihoods of the entire universe of events that might occur—the event that she prevails on the trademark claim, the event that she prevails on her defamation claim, the event that the judge is a golfer, etc.—are all represented in one huge, exhaustive, and coherent sample space in her mind, which she consults whenever she needs to estimate probabilities to inform her decisionmaking.[159]

Quantum probability models do *not* assume a single sample space.[160] Instead, quantum probability allows for multiple sample spaces.[161] Events that can be mentally represented within the same sample space and considered simultaneously are said to be "*compatible*."[162] Events that cannot be mentally represented within the same sample space and, thus, cannot be considered simultaneously, are said

153. Gorder, *supra* note 142.

154. *See id.* (quoting Professor Zheng Wang as follows: "Our brain can't store everything. We don't always have clear attitudes about things. But when you ask me a question, like 'What do you want for dinner?" [sic] I have to think about it and come up with or construct a clear answer right there,' Wang said. 'That's quantum cognition."); *see also* Busemeyer & Trueblood, *supra* note 33, at 2 (the quantum approach "provides a better representation of the *conflict*, *ambiguity*, or *uncertainty* that people experience at each moment.").

155. Gorder, *supra* note 142.

156. *Id.*

157. Bruza, Wang & Busemeyer, *supra* note 8, at 384.

158. *See supra* Section III.A.2; BUSEMEYER & BRUZA, *supra* note 21, at 6 (In classical probability theory, "[a] single sample space is proposed which provides a complete and exhaustive description of all events that can happen in an experiment."); Bruza, Wang & Busemeyer, *supra* note 8, at 384 ("[C]lassical probability theory usually assumes that events are as subsets of a single sample space.").

159. Busemeyer & Trueblood, *supra* note 33, at 3.

160. *See id.*(within a quantum framework, "[i]ncompatible questions cannot be evaluated on the same basis, so that they require setting up different sample spaces.").

161. Bruza, Wang & Busemeyer, *supra* note 8, at 384.

162. *Id.* at 385.

to be "*incompatible*."[163] This feature of quantum probability theory also parallels psychological experience: it is often difficult to entertain two events or perspectives at the same time in our minds, so we frequently have to consider them in sequence instead.[164] Put differently, incompatibility reflects limitations on our cognitive resources. The sample spaces we construct in our mind can only hold so much. Quantum decisionmaking posits that "incompatibility is ubiquitous in psychology."[165] It is generally a safe baseline assumption that people's representations of events—particularly unfamiliar events—are incompatible.[166]

For example, imagine that the plaintiff in our settlement example is prompted to estimate the joint probability that she will prevail on her trademark claim and on her defamation claim. To do so, she will likely need to consider the claims sequentially, one at a time, judging the probability for each. In the parlance of quantum decisionmaking, this would reflect incompatible events—the plaintiff mentally consults a separate "sample space" for each event. For the sake of comparison, imagine that you are asked to estimate the joint probability that the next vehicle you see will be red and that the next vehicle you see will be an SUV. This is likely an example of compatible events: if you are an experienced driver, you might well be able to draw on your experience to make one mental estimate of the proportion of vehicles you see that are red SUVs, rather than separately estimating the proportion of red vehicles and the proportion of vehicles that are SUVs.

Commutativity. Finally, and perhaps most importantly, the *commutative property* necessarily applies to all calculations within a classical probability framework.[167] This means that in simple classical models, any two events can be combined and the *order in which they are combined does not matter*: the probability of the event "X and Y"

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163. *Id.*

164. *Id.* ("Placed in a psychological context . . . a person's understanding of two events, such as two different politicians or two different perspectives on a matter, requires changing from one point of view to another, and the two points of view can imply incompatibility."). This intuition is reflected in existing legal scholarship. *See, e.g.*, Edward K. Cheng, *Reconceptualizing the Burden of Proof*, 122 YALE L.J. 1254, 1262 (2013) (noting that a civil defendant "may offer multiple possible alternatives" to the plaintiff's story, "but each of these alternatives will be judged separately, not simultaneously.").

165. Peter D. Bruza, Zheng Wang & Jerome R. Busemeyer, *Quantum Cognition: A New Theoretical Approach to Psychology* 20 (2015), https://eprints.qut.edu.au/85145/3/85145.pdf (unpublished author version).

166. *See* Jennifer S. Trueblood, James M. Yearsley & Emmanuel M. Pothos, *A Quantum Probability Framework for Human Probabilistic Inference*, 146 J. EXPERIMENTAL PSYCHOL.: GEN. 1307, 1308 (2017) ("It is generally believed that experience with a particular situation, either from previous familiarity or acquired through learning, may allow events to be represented in a compatible way, whereas relatively novel situations are more likely to be represented in an incompatible way.").

167. Bruza, Wang & Busemeyer *supra* note 10, at 384-85.

will always be identical to the probability of the event "Y and X."[168] In our example, the commutative property requires that the plaintiff's estimate of her probability of prevailing on the trademark claim and on the defamation claim be equal to the plaintiff's estimate of her probability of prevailing on the defamation claim and on the trademark claim. (This may strike the reader as intuitive—both refer to the same subset, the dark gray region, of the sample space illustrated in Figure 1.)

Quantum probability theory, on the other hand, is naturally noncommutative: *order can matter.*[169] Thinking about Y first might influence thoughts about X and vice versa. As a result, the probability of combined event Y and X may not equal the probability of combined event X and Y.[170] In fact, if X and Y are *incompatible* events, the two probabilities will not be equal. Returning to our settlement example, if the plaintiff considers her probability of prevailing on the trademark claim *before* considering her probability of prevailing on the defamation claim, then she will make *different probability judgments*—and may consequently make a different settlement decision—than she would if she considered the two claims in the reverse order, or considered them simultaneously as classical probability theory assumes.[171]

Together, these assumptions of quantum probability theory—indefinite states, the possibility of multiple sample spaces, and non-commutativity—allow the researchers using it to build a new breed of human decision-making models.[172] In many cases, these models imply that decision makers will make different probability judgments and choose different alternatives than do rational choice models.[173] As we discuss below, quantum models of decisionmaking can account for a variety of

168. *Id.* at 387. This is true even when the events occur at different times. *See also id.* at 384-85. (explaining that, in classical probability theory, "the intersection of events is always defined and events always commute, even if the events are distinguished by time (e.g., 'A at time 1' and 'B at time 2' is equivalent to 'B at time 2' and 'A at time 1')."). It is possible to model order effects with a classical model; however, it involves the inclusion of hidden variables, which are often unjustifiable. *See* Trueblood & Busemeyer, *supra* note 33, at 1519 (noting that "heuristic models lack an axiomatic foundation and only provide an ad hoc explanation for order effects.").

169. Busemeyer & Trueblood, *supra* note 33, at 2 (noting that a quantum probability theory was developed to account for non-commutative measurements); Wang et al., *supra* note 21 (illustrating how order effects that violate commutativity can be explained within a quantum framework).

170. Bruza, Wang & Busemeyer, *supra* note 8, at 384-85.

171. *Id.*

172. *Id.* Quantum models of decisionmaking have been described as "a quantized version of expected utility theory, which replaces the latter's either/or Boolean logic with the both/and logic of quantum probability theory." WENDT, *supra* note 25, at 4.

173. *See, e.g.*, *infra* Section III.B.

the heuristics and biases at the heart of BLE.[174] Indeed, quantum models have even predicted, *a priori*, previously-unobserved patterns within those "irrationalities."[175]

At this point, the reader may ask: if quantum decisionmaking researchers are adopting this peculiar quantum framework to model judgments and decisions, does that mean that they assume that the brain *physically* uses quantum processes to make judgments and decisions? This is a fair question, and the answer is "no." Quantum decisionmaking does *not* entail any claim that the brain is a quantum computer,[176] and this is what distinguishes it and its parent field "quantum cognition"[177] from the fields of "quantum brain" and "quantum mind."[178] Research in the latter fields is expressly built on the premise that the physical operations of the brain involve quantum mechanical phenomena.[179] Researchers in quantum decisionmaking, however, use quantum models as algorithms, not making any claim that they reflect

174. *See supra* Part III.B; *infra* Part IV.A; *see also* WENDT, *supra* note 25, at 4 ("Quantum decision theory predicts most of the deviations from rational behavior found by Daniel Kahneman, Amos Tversky, and others using expected utility theory as a baseline . . . ."); *cf.* David Waldner, *Schrodinger's Cat and the Dog That Didn't Bark: Why Quantum Mechanics Is (Probably) Irrelevant to the Social Sciences*, 29 CRITICAL REV. 199, 230 (2017) (arguing that quantum mechanics may not prove useful in understanding the macroscopic world, but stating that "it would be a huge mistake to confidently dismiss quantum decision theory as a source of great insight, perhaps with revolutionary consequences.").

175. *See* Wang et al., *supra* note 21 (predicting, *a priori*, a quantum pattern in order effects). This feature of quantum decisionmaking is discussed in more detail in Yearsley & Trueblood, *supra* note 22 (findings were consistent with a constraint on order effects derived, *a priori*, from quantum principles); *cf.* Owen D. Jones, *The Evolution of Irrationality*, 41 JURIMETRICS 289, 291 (2001) (Those advocating for revising the rational choice model will have a good argument if they "both identify[] and explain[] patterns in which [irrationality] arises.").

176. Busemeyer et al., *supra* note 26, at 193 ("[W]e would like to point out that we are not claiming the brain to be a quantum computer; rather, we only use quantum principles to derive cognitive models and leave the neural basis for later research.").

177. *See supra* note 101. For an overview of the quantum cognition research program, *see* Emmanuel M. Pothos et al., An Overview of the Quantum Cognition Research Programme (2018), https://pdfs.semanticscholar.org/3463/ccfd2082daa4dcaf50e697e5a4045ec52fb8.pdf [https://perma.cc/83KS-TF9S]. The field we refer to as "quantum cognition" in this paper is also sometimes described using the term "*quantum-like*" to stress the lack of claims about the underlying physical basis. *E.g.*, Andrei Khrennikov et al., *Quantum Probability in Decision Making from Quantum Information Representation of Neuronal States*, 8 SCI. REPORTS, at 1, 2 (2018), https://www.nature.com/articles/s41598-018-34531-3.pdf [https://perma.cc/CTK8-MUJA] ("To distinguish this operational approach from the approaches based on quantum physical processes in the brain . . . we call it '*quantum-like*.'").

178. *See* Yearsley & Busemeyer, *supra* note 100, at 100-01 (distinguishing quantum cognition models from "quantum brain" models); Trueblood & Busemeyer, *supra* note 31, at 1519, n.1 (same).

179. *See, e.g.*, Scott Hagan, Stuart R. Hameroff & Jack Tuszynski, *Quantum Computation in Brain Microtubules? Decoherence and Biological Feasibility*, 65 PHYS. REV. E 1; Stuart Hameroff & Roger Penrose, *Conscious Events as Orchestrated Spacetime Selections*, 3 J. CONSCIOUSNESS STUD. 36 (1996); Sultan Tarlaci & Massimo Pregnolato, *Quantum Neurophysics: From Non-living Matter to Quantum Neurobiology and Psychopathology*, 103 INT'L J. PSYCHOPHYSIOLOGY 161 (2016).

the underlying physical processes occurring in the brain but rather "leav[ing] the neural basis for later research."[180] This is similar to how rational choice theorists use rational choice models as algorithms, making no assumptions about the underlying physical processes in decision makers' brains.[181]

This Part has described, on a conceptual level, the key features that distinguish quantum models of decisionmaking from rational choice models. In Part III.B, we illustrate how these concepts are implemented in constructing a quantum model of a legal decision. We note that, depending on the reader's purposes, Part III.B may not be mandatory reading. If the reader is interested in the math underlying quantum decisionmaking or in constructing quantum models, then Part III.B (together with the Technical Appendix) provides a useful primer. But if the reader is less interested in the underlying math and more interested in broader implications of quantum decisionmaking for law and policy, then the Article is structured such that the reader can proceed to Part IV.

B. Illustrating a Quantum Model of Juror Judgment

In Part III.A, we described some of the distinct features of quantum decisionmaking. But how do these features translate to models of people's judgments? And how do these models account for heuristics and biases? This Part uses an example of a quantum model to provide introductory answers. The Technical Appendix provides more detail on how the model was constructed.

Imagine a juror is reaching a judgment in the murder trial of a previously-convicted felon (an example we previously discussed in Part II in relation to Federal Rule of Evidence 404). We are interested in the juror's assessment of the probability the defendant committed the murder[182] and, specifically, how it is influenced by the information that the defendant was previously convicted of a felony. Recall that, from a classical probability perspective, at any given moment we can represent the entire universe of the juror's probability judgments in one big

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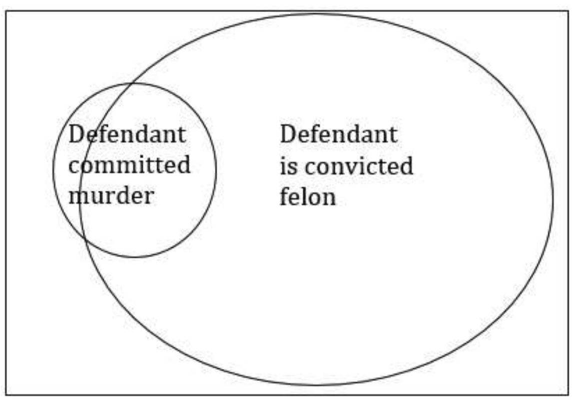
180. Trueblood & Busemeyer, *supra* note 31, at 1519; *see also* Khrennikov et al., *supra* note 177, at 2 ("As is often the case with cognitive models, in this approach the brain is considered as *a black box* that processes information in accordance with the laws of quantum information theory and generates [quantum probability] data.").

181. *See, e.g.*, Wright & Ginsburg, *supra* note 8, at 1040 ("As has long been observed, the assumption of rationality in price theory is not meant to characterize the actual decisionmaking process of economic agents.").

182. Legal scholars have long been interested in how jurors use evidence to make or update probabilistic judgments about defendants' guilt. *See generally, e.g.*, John Kaplan, *Decision Theory and the Factfinding Process*, 20 STAN. L. REV. 1065 (1967); David Kaye, *The Laws of Probability and the Laws of the Land*, 47 U. CHI. L. REV. 34 (1979); Cheng, *supra* note 165. However, this Article is, to our knowledge, the first law review article to consider the possible role of quantum probability theory in such judgments.

Venn diagram of a sample space. This sample space includes the juror's current judgments about the probability that the defendant committed murder (say it is 0.16) and about the likelihood the defendant is a previously-convicted felon (say it is 0.80; see Figure 2). According to classical probability theory, if the juror learns, definitively, that the defendant is a convicted felon, the juror updates his or her beliefs about the defendant committing murder pursuant to a specific rule. This rule is that the probability that the defendant committed murder *given that* the defendant is a convicted felon is equal to the probability of the defendant *both* being a murderer and a convicted felon (the area of the overlap of the two circles in Figure 2, assume it is 0.14) divided by the probability of the defendant being a convicted felon (the area of the "Defendant is a convicted felon" circle in Figure 2, which, again, is 0.80).[183] Thus, using the numbers in our example, the juror's updated belief would be that there is a 17.5% chance the defendant committed the murder (0.14 / 0.80 = 0.175).

Figure 2. Sample space for juror's judgments. The square repre-



sents the sample space and the circles represent the event that the defendant committed murder and the event that the defendant is a previously-convicted felon. The overlap of the two circles represents the joint event in which the defendant both committed murder and is a previously-convicted felon.

But, if the juror makes his or her probability judgments in accordance with quantum probability theory, rather than classical probability theory, a Venn diagram cannot capture all of the judgment-relevant

183. *See supra* Section III.A.2 (this rule was the third bullet point in our discussion of classical probability theory above).

information.[184] Rather than a Venn diagram, a quantum model represents the juror's judgments geometrically using a unit circle (a circle with a radius of 1 unit, see Figure 3). The unit circle contains one pair of perpendicular vectors[185] (called "basis vectors") for each relevant, incompatible[186] judgment the juror might make. In Figure 3, one pair of basis vectors (in black) represents the events "the defendant committed murder" and "the defendant did not commit murder," and the other pair (in gray) represents the events "the defendant is a previously-convicted felon" and "the defendant is not a previously-convicted felon." Each pair of basis vectors represents a *separate sample space*—reflecting that the juror cannot simultaneously represent the probability that the defendant is a murderer and the probability that the defendant is a previously-convicted felon.[187] In addition, there is one other vector in the unit circle, called the "belief state," which represents what the juror is currently thinking. On a general level, you can think of it as a needle pointing toward the juror's beliefs: the closer the belief state is to a basis vector, the more probable the juror finds event represented by that basis vector.[188]

To calculate the precise probability that the juror assigns to any particular event, we "project"[189] the belief state vector onto the basis vector corresponding to that event.[190] Functionally, this means we draw a line from the end of the belief state vector to the relevant basis vector, such that the line forms a perpendicular intersection with the basis vector (see Figure 3—the dotted lines are "projections").[191] The

184. Given that the juror is probably not particularly experienced with making legal judgments nor familiar with crime statistics, we assume that the juror's probability judgments rely on *incompatible* representations. *See supra* Part III.A.

185. In geometric terms, a "vector" is a line with a specified direction (meaning it *starts* at Point A and *ends* at Point B). *Vector*, THE CONCISE OXFORD DICTIONARY OF MATHEMATICS (5th ed. 2014).

186. A compatible quantum model will always lead to the same result as a classical probability model. *See, e.g.*, Pothos & Busemeyer, *supra* note 35, at 2 (noting that "the predictions between [classical probability] theory and quantum probability] theory with compatible questions would be identical.").

187. This reflects our assumption that the juror's representations of the two events are incompatible.

188. Yearsley & Busemeyer, *supra* note 100, at 101 ("The second ingredient in any quantum model is the specification of the initial knowledge state of a participant, or group of participants," which is captured in quantum models by the belief state); Trueblood & Busemeyer, *supra* note 31, at 1522-1523 (discussing how quantum models represent belief states with a vector).

189. Bruza, Wang & Busemeyer, *supra* note 8, at 386. For a detailed discussion of projection, *see* BUSEMEYER & BRUZA, *supra* note 21.

190. BUSEMEYER & BRUZA, *supra* note 21; *see also* Trueblood & Busemeyer, *supra* note 31, at 1523; Busemeyer et al, *supra* note 26, at 195.

191. BUSEMEYER & BRUZA, *supra* note 21, at 15-16, 31-32. One helpful analogy is that "[p]rojection is akin to shining a light from above and seeing the length of the shadow onto the plane." Bruza, Wang & Busemeyer, *supra* note 8, at 386.

result, called the "projected belief state," is a vector that begins at the center of the unit circle and ends at the point of the perpendicular intersection (e.g., the thick black line in the left-hand panel of Figure 3). The probability that the juror assigns to an event is equal to the length of the projected belief state, squared.[192] In the left-hand panel of Figure 3, the length of the projected belief state is 0.4, and the squared length of the projected belief state is 0.16. This means that, at the moment, the juror believes there is a 16% chance that the defendant committed the murder, just as there was in Figure 2. If we were to instead project the juror's belief state onto the basis vector corresponding to the event that "the defendant is a previously-convicted felon," we would see that the juror believes there is an 80% chance that the defendant is a previously-convicted felon. Thus, the juror's current belief state in the quantum model in Figure 3 is the same as the juror's current belief state reflected in the classical probability model in Figure 2.

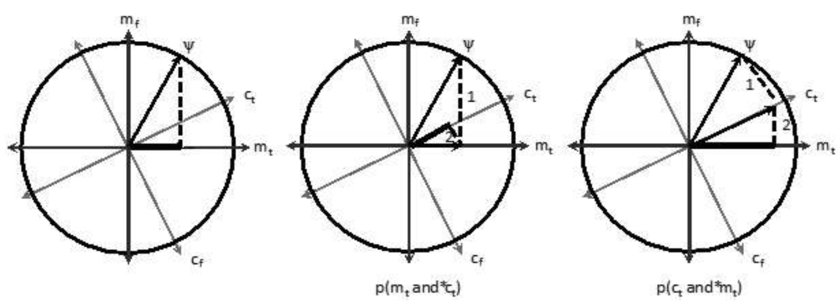


Figure 3. Constructing a quantum model of two juror judgments. The model includes two pairs of basis vectors, one pair for the juror's judgment about murder (m_f = the defendant did not commit murder; m_t = the defendant did commit murder) and one pair for the juror's judgment about whether the defendant is a previously-convicted felon (c_f = the defendant is not a convicted felon; c_t = the defendant is a convicted felon). The belief state vector is labeled with the Greek letter psi, ψ. The left panel illustrates a projection for judging the probability that the defendant committed murder (m_t). The middle panel illustrates the projection sequence for judging the probability that the defendant committed murder (m_t) followed by judging the probability that the defendant is a previously-convicted felon (c_t). The right panel illustrates the reverse order of judgments.

192. BUSEMEYER & BRUZA, *supra* note 21, at 31; Bruza, Wang & Busemeyer, *supra* note 8, at 386.

A critical feature of quantum models, however, is that the belief state does not "reset" after each projection: for models involving multiple judgments, each projection starts where the previous one ended, reflecting the juror's updated belief state (see the middle and right-hand panels of Figure 3). In our example, the starting point for calculating the juror's second judgment will be the projected belief state that results from his or her first judgment. Thus, the order of the judgments *matters*: the probability that the defendant is a murderer and a previously-convicted felon (about 0.10, illustrated by the middle panel of Figure 3) is *not equal to* the probability that the defendant is a previously-convicted felon and a murderer (about 0.64, illustrated in the right-hand panel of Figure 3).[193] This illustrates the *non-commutativity* of quantum models. Quantum models naturally give rise to "order effects"—circumstances in which judgments are affected by the order in which they are made. Order effects are difficult to reconcile with rational choice models.[194]

What does the juror do if he or she learns, definitively, that the defendant is a convicted felon? To represent "givens" in quantum models, we project the belief state onto the given event and then "normalize" it (i.e., stretch the belief state vector back out to the edge of the unit circle).[195] This reflects that the juror now assesses the probability of the given event as 1 (or 100%), and the juror operates from this revised belief state when judging the probability of the second event.[196] This is illustrated in Figure 4.

193. Bruza, Wang & Busemeyer, *supra* note 8, at 384.

194. Jennifer S. Trueblood & Jerome R. Busemeyer, *A Comparison of the Belief-Adjustment Model and the Quantum Inference Model as Explanations of Order Effects in Human Inference*, 32 PROC. OF ANN. MEETING OF COG. SCI. SOC'Y 1166 (2010) ("The presence of order effects makes a classical or Bayesian approach to inference difficult."); Busemeyer et al., *supra* note 28, at 193 ("[N]oncommutative findings in cognitive psychology, such as order effects on human judgment, suggest that classical probability theory is too limited to provide a full explanation of all aspects of human cognition."); Pothos & Busemeyer, *supra* note 35, at 40 ("[C]onjunction in [classical probability] theory is commutative (order effects can arise classically, but not without e.g., a conditionalization depending on order, which is unlikely to be known a priori.").

195. *E.g.*, Trueblood & Busemeyer, *supra* note 31, at 1524.

196. *Id.*

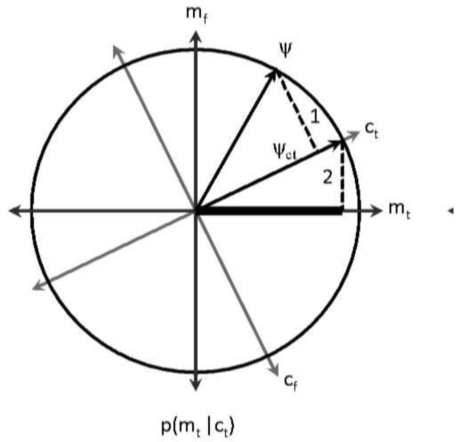


Figure 4. Quantum model illustrating representativeness / base rate neglect in judgment of the probability the defendant committed murder ($p(m_t)$) when the defendant's prior conviction is a given ($p(c_t) = 1$). The initial belief state is denoted with ψ, and the revised and normalized belief state is denoted with ψ_{ct}.

Specifically, Figure 4 illustrates a juror's judgment of the probability that the defendant committed murder *given* that the defendant is a previously-convicted felon. The probability is equal to the squared length of the thick black line (which equals 0.75)—thus, the juror believes there is a 75% chance the defendant committed the murder. This updated belief is much different than the updated belief implied by classical probability theory (which was a 17.5% chance the defendant committed the murder). Critically, in this quantum model, base rates never factor into the juror's probability assessment. Thus, our quantum model captures base rate neglect—the phenomenon scholars have identified as justification for Federal Rule of Evidence 404.[197]

In addition, the model depicted in Figure 4 naturally predicts another frequently-observed irrationality called the inverse fallacy—the tendency of people to incorrectly equate the conditional probability they are trying to judge with its inverse.[198] Our juror would commit the inverse fallacy if he or she equated the probability that the defendant is a murderer given that the defendant is a previously-convicted felon with the probability that the defendant is a previously-convicted felon given that the defendant is a murderer. If you perform the relevant

197. *See* Korobkin & Ulen, *supra* note 4, at 1086-87.

198. *See* David M. Eddy, *Probabilistic Reasoning in Clinical Medicine: Problems and Opportunities*, *in* JUDGMENTS UNDER UNCERTAINTY: HEURISTICS AND BIASES 249-67 (Daniel Kahneman, Paul Slovic & Amos Tversky, eds., 1982).

projections in the reverse order you can see this is the case in the quantum model.[199]

More detail about the construction of this quantum model (and quantum models more generally) is available in the Technical Appendix. But the critical point is that, in quantum probability theory, events are represented differently than they are in classical probability theory, and, consequently, quantum models entail a different logic than rational choice models.[200] In short, quantum models relax some of the assumptions of classical probability theory. Relaxing these assumptions allowed our example model to account for order effects, the representativeness heuristic / base rate neglect, and the inverse fallacy. In the next Part, we discuss how this quantum approach accommodates multiple other heuristics and biases discussed in legal scholarship, and identify some predictions quantum decisionmaking yields for law.

IV. LEGAL IMPLICATIONS OF QUANTUM DECISIONMAKING

Part III introduced a new psychological approach to human decisionmaking. A growing body of empirical evidence suggests that, at least in some circumstances, people "think quantum"—that their probability judgments may be better described by quantum probability theory than by classical probability theory. What does this mean for law?

Quantum decisionmaking is important to legal thinkers for (at least) two broad reasons. First, quantum decisionmaking provides a theoretical complement to BLE. BLE has been criticized on the grounds that it provides a list of exceptions to rational choice theory but does not specify what these exceptions have in common.[201] Quantum decisionmaking provides grist for a response, as quantum researchers have begun connecting many of the puzzling heuristics and biases at the heart of BLE using one coherent theoretical framework.[202]

Quantum decisionmaking also highlights the importance of sequence in choice architecture, generating a variety of new hypotheses about legal judgments and decisions. Part IV.B describes eight quantum predictions that are relevant to law and policy. Predictions such as these can lay the groundwork for new lines of legal and behavioral research, and perhaps eventually—depending on empirical findings—new policy prescriptions.

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199. This is only true by necessity in two-judgment models.

200. Busemeyer & Trueblood, *supra* note 33, at 3.

201. *E.g.*, Jones, *supra* note 52, at 1157-58; Posner, *supra* note 15, at 1574; Arlen, *supra* note 11, at 1768.

202. *See infra* Part IV.A; Yearsley & Trueblood, *supra* note 20, at 1524 ("The value of quantum models lies in their unification of disparate nonnormative behaviors.").

A. Quantum Decisionmaking as a Unifying Framework

BLE is largely an effort to correct for the shortcomings of rational choice theory.[203] Specifically, rational choice theory does not account for a number of observed heuristics and biases. BLE does, but only on an ad hoc basis.[204] That is, the *modus operandi* of BLE is to identify a previously-observed irrationality that has implications for a specific, legally-relevant context, then develop a policy recommendation that addresses or accounts for that irrationality.[205] But, without identifying any deeper pattern connecting these irrationalities, the prospects of devising comprehensive strategies for addressing them, or of predicting where new ones may arise, are slim.[206] A critic might view the undertaking as an empirically-driven game of whack-a-mole.

Quantum decisionmaking can help. Decisionmaking models grounded in quantum probability theory naturally produce many of the phenomena known as heuristics and biases. To date, these include the representativeness heuristic / base rate neglect,[207] the anchoring heuristic,[208] subadditivity,[209] both the conjunction and disjunction fallacies,[210] the availability heuristic,[211] order effects among judgments and decisions,[212] and violations of the classical law of total probability.[213] Further, quantum models naturally give rise to order effects in

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203. *See supra* Part II, and articles reviewed therein.

204. *See* Chetty, *supra* note 14, at 25 ("A common criticism of behavioral economics is that it does not offer a single unified framework as an alternative to the neoclassical model.").

205. *See, e.g.*, Cass R. Sunstein, *Nudges.gov: Behavioral Economics and Regulation*, in OXFORD HANDBOOK OF BEHAVIORAL ECONOMICS AND THE LAW 726 (2014) ("It is true that if [behavioral economics] findings are taken as a whole and in the abstract, they will not lead to a clear or unique prediction about behavior. Particular situations must be investigated in detail in order to understand likely outcomes."). Some scholars argue this is exactly what BLE *should* be—that empirical legal scholarship should strive for situation-specific models of behavior rather than a broad theory of legal judgment. *See, e.g.*, Gregory Mitchell, *Mapping Evidence Law*, 2003 MICH. ST. L. REV. 1065, 1066 n.2 (2003).

206. *See* Jones, *supra* note 52, at 1158 (advancing similar critiques relating to behavioral law and economics' lack of theoretical guidance); Kelman, *supra* note 17 (characterizing behavioral law and economics as a series of counterstories rather than a useful alternative theory).

207. Busemeyer et al., *supra* note 27, at 193.

208. Trueblood & Busemeyer, *supra* note 31.

209. Jerome R. Busemeyer, Ricardo Franco & Emmanuel M. Pothos, *Quantum Probability Explanations for Probability Judgment 'Errors'*, COGSCI (forthcoming 2018), https://arxiv.org/pdf/0909.2789.pdf [https://perma.cc/65R6-NA9H].

210. Busemeyer et al, *supra* note 27; Busemeyer et al., *supra* note 31.

211. Riccardo Franco, *supra* note 33.

212. Wang et al., *supra* note 21; Yearsley & Trueblood, *supra* note 20.

213. Bruza, Wang, & Busemeyer, *supra* note 10, at 390-91 ("Quantum theory . . . uses the principle of incompatibility to provide a straightforward account of violations of the law of total probability."); *see also* Jerome R. Busemeyer, Zheng Wang & Ariane Lambert-Mogiliansky, *Empirical Comparison of Markov and Quantum Models of Decision Making*, 53 J. MATHEMATICAL PSYCHOL. 423 (2009).

attitudes, inferences, and causal reasoning.[214] To our knowledge, this group of "irrational" phenomena had not previously been formally (mathematically) accommodated within any coherent theoretical framework.[215]

Many of the listed phenomena have played prominent roles in BLE literature, with scholars contending that they explain features of the law or warrant changes to the law. Consider the first three examples listed. The first example, the representativeness heuristic, has been used as a justification for Federal Rule of Evidence 404's prohibition of character evidence, as discussed above.[216] For example, without Rule 404, jurors might systematically overweigh evidence of prior convictions (because having prior convictions fits their stereotype of a murderer).[217]

The second example, the anchoring heuristic, is often the basis for behavioral critiques of "damage caps"—strict limits on the amounts of damages that victorious plaintiffs can recover for particular types of lawsuits.[218] Assuming that policymakers' goal in enacting damage caps is to reduce damage awards, the caps seem like a sound approach from a rational choice perspective: awards that would have been over the cap are effectively limited by the cap, and, under the assumptions of rational choice theory, the cap amounts should not affect jury awards that are under the cap.[219] But damage caps may have unintended effects: for example, if a juror is aware of an applicable damage cap, it

214. Wang et al., *supra* note 21; Yearsley & Pothos, *supra* note 35; Wang & Busemeyer, *supra* note 31. Indeed, quantum modeling has also yielded novel (and correct) *a priori* predictions about order effects in multiple contexts. *E.g.*, Wang & Busemeyer, *supra* note 33.; Yearsley & Trueblood, *supra* note 20.

215. *See generally Ashtiani & Azogmi*, *supra* note 23, at 50-51 ("The problem with all of these [earlier] approaches was that they could explain only a subset of the [decisionmaking] paradoxes and integrating them into a unified theory of decision making seemed very hard (if not impossible). Recently, a new hope for finding a unified theory of decision making that can take into account the irrationalities, paradoxes, uncertainty, ambiguity, and risk of human decision making has been found. This new hope is created as a result of applying the mathematics of quantum theory to the fields of cognition and decision making.").

216. *See supra* Part II; Korobkin & Ulen, *supra* note 4, at 1087.

217. *See* Korobkin & Ulen, *supra* note 6, at 1087.

218. Linda Babcock & Greg Pogarsky, *Damage Caps and Settlement: A Behavioral Approach*, 28 J. LEGAL STUD. 341, 355-56, 368-69 (1999); *see also* Jennifer K. Robbennolt & Christina A. Studebaker, *Anchoring in the Courtroom: The Effects of Caps on Punitive Damages*, 23 LAW & HUM. BEHAV. 353, 355 ("caps on punitive damages may trigger the anchoring and adjustment heuristic.") (1999); Michael J. Saks et al., *Reducing Variability in Civil Jury Awards*, 21 LAW & HUM. BEHAV. 243 (1997).

219. *See generally* Greg Pogarsky & Linda Babcock, *Damage Caps, Motivated Anchoring, and Bargaining Impasse*, 30 J. LEGAL STUD. 143, 159 (2001) (noting that the authors "identified the subtler effects of [damage cap] policy by expanding the traditional rational choice approach to problems of this nature.").

alters the structure of his or her decision by providing an initial number for him or her to consider—and, perhaps, unduly "anchor" on.[220] Indeed, there is empirical evidence that damage caps can function as anchors,[221] having the counterintuitive effect of pulling the amount jurors or judges award to plaintiffs in lower-value cases *up* toward the cap.[222] Perhaps, then, we should think twice about damage caps as a policy mechanism for reducing awards, or at least undertake procedures to blind jurors to damage caps in cases where jurors are awarding damages.[223]

The third example, subadditivity, occurs when one judges that the probability of a particular event is less than the probability implied by judgments of the event's subcategories or components, taken together.[224] For example, research participants tend to judge the probability of an unnatural death as lower than the same participants' estimates of enumerated *subcategories* of unnatural deaths (e.g. homicide, drowning, car accidents) would imply.[225] Professors Craig R. Fox and Richard Birke observed that lawyers might exploit this tendency in court—that jurors might find an event more likely (and assertions that it happened more credible) if lawyers "unpacked [the event] into a disjunction of constituent scenarios."[226] From a BLE perspective, one

220. The anchoring heuristic is the tendency for people to be unduly influenced by irrelevant numbers, or "anchors," when estimating an amount or value, moving their responses toward the anchor. Tversky & Kahneman, *Judgment under Uncertainty, supra* note 13, at 1128.

221. Robbennolt & Studebaker, *supra* note 218, at 361-62, 366-70.

222. William P. Gronfein & Eleanor DeArman Kinney, *Controlling Large Malpractice Claims: The Unexpected Impact of Damage Caps*, 16. J. HEALTH POL., POL'Y & L. 441 (1991); Jeffrey J. Rachlinksi, Andrew J. Wistrich & Chris Guthrie, *Can Judges Make Reliable Numeric Judgments? Distorted Damages and Skewed Sentences*, 90 IND. L.J. 695 (2015); Robbennolt & Studebaker, *supra* note 218, at 353; Michael J. Saks et al., *supra* note 220.

223. *See* Michael. S. Kang, *Don't Tell Juries About Statutory Damage Caps: The Merits of Nondisclosure*, 66 U. CHI. L. REV. 469 (1999). While many statutes that include caps also include provisions prohibiting disclosure of the caps to juries, other statutes expressly permit disclosure, and still others are silent, effectively leaving the disclosure decision to the courts. *Id.* at 471-72, 493. Some have argued that jurisdictions that do not disclose applicable damage caps should reverse course and disclose. *See, e.g.*, B. Michael Dann, *Jurors and the Future of "Tort Reform,"* 78 CHI.-KENT L. REV. 1127 (2003); Shari Seidman Diamond & Jonathan D. Casper, *Blindfolding the Jury to Verdict Consequences: Damages, Experts, and the Civil Jury*, 26 LAW & SOC'Y REV. 513 (1992).

224. Avishalom Tor & Dotan Oliar, *Incentives to Create Under a "Lifetime-Plus-Years" Copyright Duration: Lessons from a Behavioral Economic Analysis for Eldered v. Ashcroft*, 36 LOY. L. A. L. REV. 437, 465 (2002).

225. *Id.* (citing relevant psychological studies).

226. Craig R. Fox & Richard Birke, *Forecasting Trial Outcomes: Lawyers Assign Higher Probability to Possibilities That Are Described In Greater Detail*, 26 LAW & HUM. BEHAV. 159, 168 (2002). For example, a landlord's lawyer arguing that a tenant's application was "lost in the mail" might enhance credibility by unpacking the assertion into scenarios such as "the letter could have been improperly addressed, there might not have been enough postage, it might have been accidentally shredded by or stuck behind a sorting machine, it may have

might advocate rules, practices, or instructions that would mitigate such potential exploitation. Fox and Birke also found evidence that lawyers fall prey to subadditivity in evaluating case outcomes.[227] They suggested that mediators, arbitrators, and savvy judges presiding over settlement conferences could leverage this tendency to promote settlement[228]— another arguably-exploitative practice that reasonable lawyers could argue for or against.

In considering these three examples—representativeness, anchoring, and subadditivity—the reader may or may not find BLE policy arguments to be persuasive. But regardless of the merits of each argument, the reader likely interpreted them as three *independent* arguments. Generally speaking, that is how BLE arguments have been presented.[229] This is because, from a BLE perspective, the behavioral phenomena underlying policy arguments are generally thought of as heuristics and biases, meaning they are *exceptions* to the generally-applicable decision-making rules of rational choice.[230] And, on this view, capturing the effects of each of these heuristics on human decisionmaking requires a unique, heuristic-specific and context-specific model (e.g., a model reflecting the representativeness heuristic in juror decisions may be built on a different set of assumptions than a model reflecting the anchoring heuristic in juror decisions or than a model

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been dropped by a letter carrier, or it could have been lost in the mail for some other reason." *Id.*

227. In a hypothetical negligence case, lawyers found that potential problems with proving "duty, breach, or causation" to be a bigger problem than potential problems with proving "liability," even though "all lawyers understand that tort liability consists of duty, breach, and causation." *Id.* at 167. Further, this manifestation of subadditivity affected the lawyers' advice on whether to settle hypothetical cases. *See id.* ("If the junior colleague was concerned with meeting the standard on 'liability,' roughly half the participants (52%) recommended accepting the settlement offer. However, if she was concerned with meeting the standard on 'duty, breach, or causation,' a pronounced majority of participants (74%) recommended accepting the settlement offer.").

228. *See id.* ("[A] neutral hoping to produce settlement could exploit subadditivity by selectively unpacking possible unfavorable outcomes in private discussions with each party[,] . . . potentially increase[ing] both sides' perception of an unfavorable outcome and therefore increase[ing] their willingness to settle.").

229. *See Wright & Ginsburg*, *supra* note 8, at 1040 (noting that behavioral law and economics has struggled to develop a "comprehensive theory of errors," instead "focus[ing] largely upon the effort to catalog circumstances in which economic decisionmakers appear systematically to depart from rational choice behaviors.").

230. *See* Jones, *supra* note 3, at 490 (noting that heuristics and biases have been presented as "a category defined more by what they are not than by what they are"); Arlen, *supra* note 11, at 1768 ("Behavioral analysis of law does not have a coherent model of human behavior in part because . . . behavioral economists and cognitive psychologists generally have focused on demonstrating that people do not necessarily exhibit rational choice."); *see also* Bruza, Wang, & Busemeyer, *supra* note 10, at 387 (arguing that, thus far, violations of classical models of human decisionmaking "have been explained using heuristic rules such as the representativeness heuristic and the anchoring-and-adjustment heuristic," rather than using a "coherent, common set of principles.").

considering "rational" aspects of juror decisionmaking). As a consequence, BLE has fundamentally been a piecemeal endeavor: BLE often focuses on the relationship between one observed irrationality and one legal issue at a time, with the empirical evidence supporting prescriptions typically cabined to data on the particular irrationality of focus.[231]

Many legal scholars have suggested the endeavor would be smoother—and more persuasive—if prescriptions were premised on a broader understanding of human behavior.[232]

> Given the wide variety of cognitive heuristics, biases, and quirks that have been feted as significant to human behavior, economics, and legal regimes, we should aspire not to ad hoc explanations for each, but instead to a connective theory that might make seamless and coherent sense of the phenomena as a group.[233]

Such a connective theory could provide organizing principles to help distinguish stable patterns of "irrational" decisions (that may warrant adjustments to policies developed based on assumptions of rationality) from true anomalies (that likely do not warrant policy adjustments). Further, a connective theory can help predict when heuristics and biases are likely to manifest and strengthen support for debiasing policy prescriptions. If, for example, the representativeness heuristic, anchoring, and subadditivity are different manifestations of the same underlying phenomenon, then evidence relating to all three can be brought to bear to analyze policy issues that would otherwise be informed only by one.

Quantum decisionmaking may help provide the connective theory that BLE has been said to lack. Quantum decisionmaking aims to situate "rational" and "irrational" decisions within one coherent and generally-applicable decision-making framework.[234] This framework can provide organizing principles that connect and strengthen BLE-driven policy arguments. Indeed, quantum researchers have already made

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231. *See* Jones, *supra* note 52, at 1158 ("[I]n the absence of buttressing theory such efforts represent isolated successes, rather that promisingly synergistic ones that would signal a broad, systematic approach."). As mentioned previously, some scholars argue this is as it should be. *See* Mitchell, *supra* note 205.

232. *See, e.g.*, Jones, *supra* note 175, at 291; Korobkin & Ulen, *supra* note 4, at 1057 ("This movement, which we call 'law and behavioral science,' lacks a single, coherent theory of behavior.").

233. Owen D. Jones & Sarah F. Brosnan, *Law, Biology, and Property: A New Theory of the Endowment Effect*, 49 WM. & MARY L. REV. 1935, 1952 (2008).

234. Bruza, Wang & Busemeyer, *supra* note 8, at 383.

connections between the three heuristics discussed above—representativeness, anchoring, and subadditivity.[235] From a quantum perspective, these three heuristics are simply different instantiations of incompatible representations, like three different symptoms that arise from the same underlying disease. Extending this metaphor, quantum decisionmaking suggests that the same "treatment" (i.e., the same debiasing strategies) may be effective for combating all three heuristics, to the extent policymakers decide such heuristics should be combatted. What appeared, from a traditional BLE perspective, to be three distinct inquiries—(i) when is the representativeness heuristic likely to occur and how can we correct for it?, (ii) when is the anchoring heuristic likely to occur and how can we correct for it?, and (iii) when is subadditivity likely to occur and how can we correct for it?—is, from a quantum perspective, reduced to one inquiry: When do people tend to use incompatible representations, and how can we prompt people to use compatible representations instead?[236]

B. Eight Legal Predictions

Quantum decisionmaking provides a fresh source of hypotheses about decisions that are of interest to law. Perhaps the broadest hypothesis that arises from quantum decisionmaking is that the irrationalities underlying BLE should flock together.[237] That is, in contexts where people use incompatible representations, quantum decisionmaking predicts that their decisions will show order effects, conjunction fallacies, disjunction fallacies, and a number of other irrationalities—where one irrationality manifests, others should too.[238] While scholars may have previously assumed that this is the case,[239] quantum decisionmaking provides theoretical justification for this assumption.

Quantum decisionmaking also broadly suggests that, when decision makers make a series of judgments, their judgments are likely affected by the order in which they are made.[240] This sort of path dependency

235. We demonstrated how quantum decisionmaking accounts for the representativeness heuristic in Section III.B, *supra*. *See also* Busemeyer et al., *supra* note 27 (representativeness heuristic); Trueblood & Busemeyer, *supra* note 31 (anchoring); Busemeyer, Franco, & Pothos, *supra* note 209 (subadditivity).

236. *See* Bruza, Wang & Busemeyer, *supra* note 8, at 392 ("Our view is that incompatibility of events provides an effective solution to bounded resources, which is the reason for bounded rationality.").

237. Yearsley & Trueblood, *supra* note 20 (demonstrating the co-occurrence of conjunction fallacy and order effects in political decisionmaking).

238. *E.g.*, Busemeyer & Wang, *supra* note 23, at 167.

239. *See* Chetty, *supra* note 14, at 27 ("The intuition underlying this assumption is that behavioral biases are positively correlated.").

240. Quantum models of decisionmaking naturally give rise to order effects. To be certain, other models of legal decisionmaking also predict some order effects—at least, effects

has broad legal relevance, as law is, in many respects, a mechanism for structuring judgment and decisionmaking.[241]

In this Part, we describe eight specific examples of legal predictions that arise naturally from quantum decisionmaking,[242] with implications ranging from litigation strategy to eyewitness lineup construction to police training.[243] Some of these predictions highlight instances in which decision makers' judgments are likely prone to order effects. Other predictions arise from extending the analogy between human decisionmaking and quantum physics. That is, quantum decisionmaking predicts that certain contextual influences observed in quantum physics should have analogs in human decisionmaking.[244] All of these predictions provide promising avenues for behavioral and legal research and, potentially, future policy prescriptions.

Prediction #1: Quantum effects influence factfinders' final judgments. Quantum decisionmaking predicts that changes in evidence-based opinions (such as whether a criminal defendant is guilty or innocent) will be slowed down by repeated judgments.[245] In other words, quantum principles suggest that when someone's opinion on a topic is "measured"—when she articulates it or consciously identifies it—it becomes less likely to change, even in the presence of accumulating evidence to the contrary.[246] Essentially, the person's beliefs (represented by the "belief state" in a quantum model) keep getting "reset" to reflect

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of the order in which decision makers consider evidence (e.g., the story model and coherence-based reasoning). Quantum decisionmaking is distinct, however, in that order effects arise naturally from the probability theory in which it is based. Further, quantum decisionmaking is unique in that it predicts that the order effects with respect to the order in which people make *judgments*, regardless of the order in which evidence is presented. Finally, quantum decisionmaking can yield specific predictions about signature patterns within order effects. *See, e.g.*, Wang et al., *supra* note 21 (predicting, based on quantum principles, a pattern called "QQ equality" in the order effects that participants displayed); Yearsley & Trueblood, *supra* note 20 (findings were consistent with a constraint on order effects derived, *a priori*, from quantum principles).

241. CASS. R. SUNSTEIN, BEHAVIORAL LAW AND ECONOMICS 2 (Cass. R. Sunstein ed., 2000) ("[T]he legal system is pervasively in the business of constructing procedures, descriptions, and contexts for choice.").

242. We use the word "prediction" here in the scientific sense that, working from the principles of quantum decisionmaking, we would expect to observe these phenomena. Some of them have already been observed.

243. We note that some of these eight predictions could arguably be justified using different theoretical bases (e.g., coherence-based reasoning), but all eight arise naturally from quantum decisionmaking.

244. For example, the "quantum Zeno effect" has an analog in human decisionmaking. *See infra* Part IV.B.1.

245. Yearsley & Pothos, *supra* note 35.

246. This prediction arises from the collapse postulate of quantum theory: the idea is that, whenever someone makes a decision, his or her belief state changes to reflect the outcome. *Id.* at 1.

the person's judgment each time she makes one.[247] This is analogous to the principle in quantum physics "that a continuously observed unstable particle never decays," known as the quantum Zeno effect.[248]

Professors Yearsley and Pothos recently ran two experiments as initial tests of this prediction.[249] In both, participants began by reading some basic facts of a hypothetical murder case involving a defendant named Smith. After reading these basic facts, participants were asked to make a preliminary judgment of Smith's guilt. The vast majority of participants—in excess of ninety percent across the two experiments—initially thought Smith was likely innocent.[250] Participants then read about twelve pieces of evidence suggesting Smith was guilty. Each piece of evidence was designed to be weak standing alone but, cumulatively, the evidence was relatively strong (as the researchers confirmed through pilot testing). After reading all of the evidence, participants were asked to make a final judgment about whether Smith was guilty. Critically, however, some participants were asked to make one or more additional judgments of Smith's guilt between the initial and final judgments. Specifically, participants were split into six groups, with five of the six groups making intermediate judgments of guilt (after intervals of one, two, three, four, and six pieces of evidence, respectively).

The researchers found evidence of the quantum Zeno effect: having to make more intermediate judgments tended to prevent participants from changing their initial judgment. The effect was quite dramatic, too. Of participants who started off thinking Smith was innocent, those who made twelve intermediate judgments (one after every piece of evidence) ultimately changed their mind and found Smith guilty less than ten percent of the time. The participants who only made preliminary and final judgments, on the other hand, ultimately changed their minds and found Smith guilty close to fifty percent of the time. [251] Note that the quantum Zeno effect is not an order effect in the conventional sense: it is not a product of the order in which evidence is presented,

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247. This has some parallels to the idea of "bidirectional reasoning" in coherence-based reasoning models, *see generally* Simon & Holyoak, *supra* note 86.

248. Yearsley & Pothos, *supra* note 35.

249. The facts in this paragraph and the next are drawn from *id.* at 1-6.

250. This is, perhaps, encouraging news for the health of the presumption of innocence. *Cf.* Cathleen Burnett, *Constructions of Innocence*, 70 UMKC L. REV. 971, 974 (2002) (arguing that "the practical reality of the trial process is more likely to turn this presumption [of innocence] inside out, resulting in the defendant being presumed 'guilty until proved innocent.' " (quoting RONALD C. HUFF ET AL., CONVICTED BUT INNOCENT: WRONGFUL CONVICTION AND PUBLIC POLICY 213-25 (1996)); Josephine Ross, "*He Looks Guilty": Reforming Good Character Evidence to Undercut the Presumption of Guilt*, 65 U. PITT. L. REV. 227, 228-29 (2004) (contending that the presumption of innocence has been eroded in contemporary criminal practice).

251. These figures represent an average across the two experiments.

but of the number of conscious intermediate judgments the decision makers make when considering evidence.[252]

The quantum Zeno effect has practical legal implications ranging from the structural to the tactical. One can imagine, for example, a criminal defense attorney using a part of her opening statement to ask jurors to consider each bit of the prosecution's evidence as it is presented and think about whether it persuades them that the defendant is guilty. One might also suspect that jurors with certain traits—for instance, those inclined to think deeply and re-evaluate their beliefs—might be more prone to the quantum Zeno effect, and that questions related to such traits might therefore be useful during voir dire.[253]

Finally, we note that the quantum Zeno effect is both highly "irrational" from a classical perspective and highly relevant to law, yet it is not on the traditional list of heuristics and biases discussed in the BLE literature. This is because the prediction of its existence arose from quantum decisionmaking.[254] Yearsley and Pothos's work nicely illustrates quantum decisionmaking's potential as a catalyst for uncovering new, and uniquely quantum, quirks in legal decisionmaking.[255]

Prediction #2: Eyewitnesses responding to sequentially-presented lineups demonstrate a systematic bias in their responses. Wally Witness visits a police station to identify the sole perpetrator of a crime he witnessed. The police officer has two photographs to show Wally—a picture of Dave and a picture of Mike. The officer might show Wally the first picture and ask, "Did you see Dave?" then show him the second picture and ask, "Did you see Mike?" (this is called "sequential" presentation). Alternatively, the officer might show Wally both pictures and ask, "Did you see either Dave or Mike?" This is called "simultaneous" presentation.[256] From a rationalist perspective, the officer's choice between sequential and simultaneous presentation should not

252. Yearsley & Pothos, *supra* note 35, at 6 ("We have shown here that opinion change depends not just on the evidence presented, but can also be strongly effected by making intermediate judgements, in the particular way predicted by the quantum model.").

253. Research has suggested that people with certain thinking styles are more prone to reflect and re-evaluate their knowledge and opinions than others. *See, e.g.*, D. N. Perkins, Eileen Jay & Shari Tishman, *Beyond Abilities: A Dispositional Theory of Thinking*, 39 MERRILL-PALMER Q. 1 (1993).

254. *See* Yearsley & Pothos, *supra* note 35, at 1 (prediction generated *a priori* from quantum principles).

255. *See id.*

256. *See, e.g.*, Nancy K. Steblay, Jennifer E. Dysart & Gary L. Wells, *Seventy-Two Tests of the Sequential Lineup Superiority Effect: A Meta-Analysis and Policy Discussion*, 17 PSYCHOL. PUB. POL'Y & L. 99 (2011) (contrasting "simultaneous" presentation, in which lineup members are presented all at once, with "sequential" presentation, in which lineup members are presented one at a time).

make any difference.[257] But psychological research suggests the choice of presentation style matters.

Historically, psychologists have argued that police should use sequential, not simultaneous, lineup presentations.[258] The reasoning behind this recommendation was that, when a witness views a full lineup at the same time, he or she might engage in a relative judgment process (comparing features of lineup members and choosing the member who most closely resembles his or her memory of the perpetrator) rather than an absolute judgment process (individually comparing each lineup member to his or her memory of the perpetrator).[259] Based in large part on this research, approximately thirty percent of law enforcement agencies in the United States that use photo lineups were using sequential presentation as of 2016.[260]

Quantum decisionmaking, however, casts some doubt on whether sequential presentation is preferable. Specifically, quantum decisionmaking yields an *a priori* prediction that, regardless of what the witness actually saw, the witness is more likely to indicate that he or she saw one of the lineup members in a sequential lineup than a simultaneous lineup (a phenomenon called the "episodic over-distribution effect").[261] This effect—which has been documented in a number of studies[262]—might lead to an increased number of false alarms in sequential lineups. Thus, quantum decisionmaking suggests a substantial downside to the sequential lineup procedure traditionally advocated by psychologists. Researchers are revisiting the issue,[263] and policymakers are taking note: The Department of Justice recently issued guidance that walked back its previous endorsements of sequential lineup procedures.[264]

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257. *See, e.g.*, Trueblood & Busemeyer, *supra* note 31, at 1518-19 (noting that classical Bayesian models have a difficult time accounting for situations in which the order in which information is presented affects outcomes); Bilz, *supra* note 14, at 435 ("In a Bayesian model, the order in which one presents evidence should not matter.").

258. For review and discussion, see, for example, Steve Charman & Gary L. Wells, *Applied Lineup Theory*, *in* 2 HANDBOOK OF EYEWITNESS PSYCHOLOGY 219 (Lindsay, Ross, Read & Toglia, eds., 2006); Gary L. Wells et al., *Eyewitness Identification Procedures: Recommendations for Lineups and Photospreads*, 22 LAW & HUM. BEHAV. 603, 616-17, 639-40 (1998); Steblay, Dysart & Wells, *supra* note 256.

259. Wells et al., *supra* note 258, at 613-14.

260. John T. Wixted et al., *Estimating the Reliability of Eyewitness Identifications from Police Lineups*, 113 PROC. NAT'L. ACAD. SCI S. 304, 304 (2016).

261. *See* Jennifer S. Trueblood & Pernille Hemmer, *The Generalized Quantum Episodic Memory Model*, 41 COGNITIVE SCI. 2089, 2092-94 (2017).

262. For review and meta-analysis, *see* Charles J. Brainerd & Valerie F. Reyna, *Episodic Over-distribution: A Signature Effect of Familiarity Without Recollection*, 58 J. MEMORY & LANGUAGE 765 (2008).

263. *See* Wixted et al., *supra* note 260, at 304.

264. *See* SALLY Q. YATES, MEMORANDUM TO HEADS OF LAW ENFORCEMENT COMPONENTS AND ALL DEPARTMENT PROSECUTORS, U.S. DEP'T OF JUSTICE, OFF. OF THE DEPUTY ATT'Y GEN. (2017), https://www.justice.gov/file/923201/download [https://perma.cc/B4VB-82CM]

Prediction #3: Order effects influence responses on ballots, human resources forms, and more. As noted previously, quantum decisionmaking is relevant whenever people are asked to make a series of judgments or decisions in succession. One prominent example is voting.

Quantum decisionmaking predicts that the order of items on the ballot will systematically influence results, and empirical evidence supports this prediction. For example, when pollsters asked people (i) whether Al Gore is honest and trustworthy and (ii) whether Bill Clinton is honest and trustworthy, responses varied substantially based on the order in which the questions were asked.[265] People who responded in the Gore-Clinton order were significantly more likely to say that they found Al Gore trustworthy than participants who responded in the Clinton-Gore order.[266] Order effects are most easily observed when the order of judgments or decisions is predictable (e.g., where it is likely to follow the order presented on the ballot).[267] Similar order effects are likely to occur in other form-filling situations, from consumers completing order forms to employees making insurance or investment decisions.

It should be noted that quantum decisionmaking generally does not yield *a priori* predictions about the direction or magnitude of these order effects. Rather, quantum decisionmaking simply emphasizes a dimension of choice architecture—sequence—that is likely to significantly affect decisions. This is consistent with Professor Cass Sunstein's observation about BLE more generally: "Particular situations must be investigated in detail in order to understand likely outcomes."[268]

Prediction #4: Order effects influence verdicts in cases involving multiple charges/claims or multiple defendants. In the United States, civil plaintiffs and criminal prosecutors alike may bring multiple claims, against multiple people, in one action.[269] This sometimes leaves juries or judges tasked with making a series of judgments, often in

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(noting recent research raising questions about the superiority of sequential methods and suggesting that "simultaneous procedures may result in more true identifications and fewer false ones"). Further, in 2016, the International Association of Chiefs of Police dropped its longstanding endorsement of sequential lineup procedures. *See Wixted Memory Laboratory*, U.C., SAN DIEGO, http://wixtedlab.ucsd.edu/ [https://perma.cc/7BKY-P2MB].

265. David W. Moore, *Measuring New Types of Question-Order Effects: Additive and Subtractive*, 66 PUB. OPINION Q. 80, 81-83 (2002).

266. *Id.* at 82-83.

267. *See id.* at 89-90 (noting that the order in which questions are presented on a survey can affect results—results "obtained in one order are often quite different from the results obtained in the opposite order.").

268. Sunstein, *supra* note 205, at 726.

269. *See id.* at 19-20 (joinder of parties); FED. R. CIV. P. 18 (joinder of claims); FED. R. CRIM. P. 8 (joinder of offenses or defendants).

relatively quick succession (e.g., is the defendant liable for trademark infringement? Is the defendant liable for defamation?). Further, the sequence in which jurors and judges make these decisions may be quite predictable (e.g., based on the order of the jury instructions). From a quantum perspective, this presents a set of circumstances in which we would expect any judgment in the sequence to systematically influence the following judgment(s) (similar to the Clinton-Gore polling example discussed above).[270] There is little reason to think that legal judgments of jurors (or judges, for that matter) are immune to order effects in such circumstances.[271] Empirical guidance concerning the prevalence, magnitude, and any patterns of these "verdict form order effects" might inspire policies and procedures to mitigate—or at least standardize—them.[272]

Prediction #5: Appellate courts likely overuse "harmless error" doctrine. A litigant who is unhappy with the outcome she receives at the trial court level has the option of pursuing an appeal.[273] To prevail on appeal, the dissatisfied litigant generally must show that some error occurred at the trial level.[274] But, demonstrating an error occurred does not mean that the litigant will prevail on appeal. "Harmless" errors—those that "do not affect the substantial rights of the parties"—do not justify disturbing the trial court's decision.[275] Error is deemed harmless when the appellate court concludes that it would not have affected the outcome of the proceeding below—that is, when "the factfinder most likely would have rendered the same decision based on the untainted evidence had the error not occurred."[276]

The harmless error doctrine is built on a model of decisionmaking that assumes (1) that decision makers rationally consider each piece

270. *See, e.g.*, Moore, *supra* note 265.

271. Indeed, some previous research shows order effects on certain judicial decisions. *See* Rachlinksi, Wistrich & Guthrie, *supra* note 222, at 729-31.

272. It might also inspire increased argument between litigants over verdict forms.

273. *See* Cassandra Burke Robertson, *The Right to Appeal*, 91 N.C. L. REV. 1219, 1222 (2013) ("[T]he federal court system and forty-seven states provide—as a matter of state law—either a constitutional or statutory requirement for appeals as of right in both civil and criminal cases.").

274. *See* Chad M. Oldfather, *Error Correction*, 85 IND. L.J. 49, 49 (2010) ("Most depictions of appellate courts suggest that they serve two core functions: the creation and refinement of law and the correction of error.").

275. For example, the federal harmless error statute provides that "[o]n the hearing of any appeal or writ of certiorari in any case, the court shall give judgment after an examination of the record without regard to errors or defects which do not affect the substantial rights of the parties." 28 U.S.C. § 2111; *see also* FED. R. CRIM. P. 52(a) ("Any error, defect, irregularity, or variance that does not affect substantial rights must be disregarded."); FED. R. CIV. P. 61 ("At every stage of the proceeding, the court must disregard all errors and defects that do not affect any party's substantial rights.").

276. Justin Murray, *A Contextual Approach to Harmless Error Review*, 130 HARV. L. REV. 1791, 1795-96 (2017).

of evidence independently and ultimately aggregate them to reach a legal judgment, and (2) that each of the decision makers' legal judgments is independent from all others.[277] On this model, determining whether an erroneously-considered piece of evidence affected the decision is not an especially difficult problem. An appellate court needs only factor out the value of that individual piece of evidence and "rerun the numbers" for the particular legal judgment(s) to which the evidence was relevant.[278]

Quantum decisionmaking, however, suggests that the factfinders at the trial level almost certainly do *not* weigh each piece of evidence independently[279] or make serial judgments independently. As factfinders receive new information, the decision maker's beliefs "realign[ ] with the current contents of short-term memory . . . and the perspectives that flow from those contents."[280] Thus, each piece of evidence the factfinder considers can have a systematic effect on every piece of evidence that follows it, potentially having disproportionate influence on the relevant legal judgment.[281] Further, because judgments systematically affect subsequent judgments,[282] the effects of erroneously-considered evidence could ripple out to legal judgments relating to other charges or even other defendants.

Because it is unlikely that appellate courts fully account for these effects in conducting harmless error review, we expect that courts likely over-apply the "harmless error" construct. Researchers could test this idea empirically, assessing the effect of the presence or absence of a piece of evidence on participants' verdicts in a mock case involving multiple charges, then asking another group of participants to review the full body of evidence and assess whether excluding the

277. Jason M. Solomon, *Causing Constitutional Harm: How Tort Law Can Help Determine Harmless Error in Criminal Trials*, 99 NW. U. L. REV. 1053, 1087-88 (2005) (describing the "probabilistic" or "algebraic" model of juror decisionmaking as the "dominant paradigm in the harmless-error analyses"); Bilz, *supra* note 12, at 435 ("In Bayesian models, the judger comes to a decision by algebraically assessing the probabilities of a conclusion given various pieces of evidence. In a Bayesian model, the order in which one presents evidence should not matter . . . .") (footnote omitted).

278. A clear example of this approach can be found in the Fourth Circuit's opinion in *Cooper v. Taylor*, 103 F.3d 366 (4th Cir. 1996). In *Cooper*, the *en banc* court analogized the case to a baseball game, in which invalidating the government's "grand-slam home run" evidence would only change the final score from 14-0 in favor of the government to 10-0 in favor of the government. *Id.* at 370.

279. This prediction is also consistent with the story model and coherence-based reasoning. *See* Pennington & Hastie, *supra* note 84, at 520-21; Holyoak & Simon, *supra* note 86, at 22.

280. Wang et al., *supra* note 21, at 9434.

281. *See supra* notes 78-84; *see also* Craig R. M. McKenzie, Susanna M. Lee & Karen K. Chen, *When Negative Evidence Increases Confidence: Change in Belief After Hearing Two Sides of a Dispute*, 15 J. BEHAV. DECISION MAKING 1 (2002) (finding that jurors' weighing of a particular piece of evidence varied based on other evidence presented around it).

282. Busemeyer et al., supra note 26, at 193.

critical piece would likely affect the outcomes on each charge. Researchers could also examine whether patterns in the results of such a study reflect certain statistical patterns considered signatures of quantum decisionmaking.[283]

Prediction #6: The order in which courts consider relevant authorities may affect their legal conclusions. In a thoughtful and thorough 2016 article, Professor Samaha considers the importance of sequencing in law, focusing specifically on the sequence in which judges consider authorities in reaching legal interpretations of statutes.[284] Among other sources, Professor Samaha draws from psychological research on order effects to argue that seemingly mundane judicial dictates about analytical starting points—like "starting with the text"—might have systematic influences on legal decisions.[285] Professor Samaha observes that, while the sequence of interpretive sources a decision maker considers should not matter from a rational choice perspective,[286] research findings on topics ranging from personality assessment to memory to consumer behavior demonstrate that sequence often does matter.[287] Professor Samaha notes that theories attempting to account for these various order effects—which can include primacy effects, recency effects, and others, depending on context—have traditionally been "complicated and hedged."[288] Accordingly, their predictions for whether and how these effects might manifest for judges interpreting a statute is not clear; at most, any predictions must be contingent on a number of assumptions about the number of relevant sources, the judge's time constraints and delegation practices, the amount of time the judge will spend integrating and evaluating information between sources, and how the judge will likely go about integrating new information.[289]

Quantum decisionmaking can help to clarify these predictions. We have emphasized that order effects go hand-in-hand with quantum decisionmaking: from a quantum perspective, order effects will naturally arise any time a decision maker uses incompatible representations.[290]

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283. *See, e.g.*, Yearsley & Trueblood, *supra* note 20.

284. *See* Adam M. Samaha, *Starting with the Text—On Sequencing Effects in Statutory Interpretation and Beyond*, 8 J. LEGAL ANALYSIS 439 (2016).

285. *Id.* at 483 ("In fact, following this sequence rule easily could influence case results—perhaps counterintuitively and disturbingly so, depending on the mechanism for the effect.").

286. *Id.* at 456-57.

287. *Id.* at 461-69.

288. *Id.* at 465 (discussing summary of Hogarth & Einhorn's theoretical efforts in Robin M. Hogarth & Hillel J. Einhorn, *Order Effects in Updating: The Belief-Adjustment Model*, 24 COG. PSYCH. 1 (1992)).

289. *Id.* at 469 (predictions must be based on a number of assumptions, and if one is to "[t]inker with a few assumptions . . . the prediction disintegrates or reverses.").

290. *See supra* Part III.B.

From a quantum perspective, then, the question of whether order effects will manifest is one of compatibility. A natural baseline assumption is that different sources are likely represented as incompatible,[291] making order effects probable—though subject (as discussed in Section IV.B.7 below) to potential individual differences in proclivity for cognitive reflection and familiarity with the relevant topic and sources.[292] As to the *direction* of these order effects (i.e., will the first sources consulted or last sources consulted have more influence on decisions): here, quantum principles do not provide an *a priori* prediction, but the data suggests that the longer the series of sources considered in reaching a decision, the more likely it is that the first sources consulted will have disproportionate influence on ultimate decisions ("primacy effects").[293]

Prediction #7: Police officers who deliberately categorize suspects before engaging them might be less likely to respond to suspects with force. This prediction—which may strike the reader as exactly backwards—is based on an ongoing line of experiments on the quantum effects of categorizing on subsequent decisions.[294] In these studies, experimenters take pictures of faces and then assign each face to one of two categories (typically either a "good guy" category or "bad guy" category) based on certain features (e.g., face width).[295] Participants then view the pictures of the faces on a computer screen, presented one at a time.[296] For each face, participants are asked to do one of three things: (1) categorize the person depicted as a "good guy" or "bad guy" (categorization-only trials), (2) decide whether to "attack" or "withdraw" (decision-only trials), or (3) first categorize and then decide (joint categorization-decision trials).[297] After responding, participants receive feedback letting them know whether they made the correct choice(s)—they were generally rewarded for attacking bad guys and withdrawing from good guys.[298]

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291. *See* Bruza, Wang & Busemeyer, *supra* note 165 ("incompatibility is ubiquitous in psychology").

292. *See infra* Part V, for a discussion of the potential relevance of familiarity and cognitive reflection.

293. Hogarth & Einhorn, *supra* note 288. For a comparison of Hogarth and Einhorn's belief-adjustment model and a quantum model, *see* Trueblood & Busemeyer, *supra* note 196.

294. *See* James T. Townsend et al., *Exploring the Relations Between Categorization and Decision-making with Regard to Realistic Face Stimuli*, 8 PRAGMATICS & COGNITION 83 (2000); Zheng Wang & Jerome R. Busemeyer, *Interference Effects of Categorization on Decision Making*, 150 COGNITION 133 (2016); Busemeyer, Wang & Lambert-Mogiliansky, *supra* note 213.

295. For discussion of procedures in this line of studies, *see* Wang & Busemeyer, *supra* note 294, at 134, 136-41.

296. *Id.*

297. *Id.*

298. *Id.*

Researchers using this paradigm find, unsurprisingly, that the probability of participants "attacking" someone they have labeled a "good guy" is much lower than the probability of participants "attacking" someone they have labeled as a "bad guy."[299] But another finding, based on comparing the joint categorization-decision trials to the decision-only trials, is much more surprising: across all studies to date, participants choose to attack a "bad guy" who has been categorized as a "bad guy" less frequently than they choose to attack the exact same "bad guy" in decision-only trials (where they make no categorization decision at all).[300] That is, it seems that categorizing—even categorizing *as a "bad guy"*—makes participants *less likely* to choose to "attack."[301]

The idea that categorizing people can reduce one's impulse to attack is not at all intuitive, but it is predicted by quantum theory.[302] If this pattern persists in more realistic settings, it could have important real-world implications. Consider the unfortunate and alarming rate at which stories of police officers responding violently to black suspects have appeared in the news lately.[303] One of the strange potential implications of these categorization studies is that, the more a police officer deliberately and consciously categorizes the suspects he encounters, the less likely he may be to respond to those suspects with hostility, even if he categorizes them in a negative light. If evidence of this surprising effect of categorization continues to mount and replicates in more naturalistic situations, it might one day have applications in police training (e.g., categorize first, engage second).

We, of course, are making no recommendation of the sort at this time. There is an enormous difference between participants hitting

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299. *Id.*

300. *Id.*; *see also* Townsend et al., *supra* note 294; Busemeyer, Wang & Lambert-Mogiliansky, *supra* note 213.

301. Wang & Busemeyer, *supra* note 294, at 134, 145 (describing effects found in previous studies and reporting effects replicated in the authors' latest studies).

302. *Id.* at 145-46 (discussing success of quantum model in accounting for interference effects relative to other models).

303. *See, e.g.*, Christopher Brennan, Nicole Hensley & Denis Slattery, *Alton Sterling Shot, Killed by Louisiana Cops during Struggle after He Was Selling Music Outside Baton Rouge Store*, N.Y. DAILY NEWS (July 6, 2016), https://www.nydailynews.com/news/national/la-cops-shoot-kill-man-selling-music-baton-rouge-store-article-1.2700548 [https://perma.cc/MD4L-VKFX]; Ciara McCarthy, *Philando Castile: Police Officer Charged with Manslaughter over Shooting Death*, THE GUARDIAN (Nov. 16, 2016), https://www.theguardian.com/us-news/2016/nov/16/philando-castile-shooting-manslaughter-police-jeronimo-yanez [https://perma.cc/EM5A-HY2R]; Jason Meisner, *The Lingering Questions in Laquan McDonald Shooting Case*, CHI. TRIBUNE (DEC. 5, 2015), https://www.chicagotribune.com/news/ct-chicago-cop-shooting-laquan-mcdonald-faq-met-20151204-story.html [https://perma.cc/2YRX-GQ4V]; Leah Thorsen & Steve Giegrich, *Officer Kills Ferguson Teen*, ST. LOUIS POST-DISPATCH (Aug. 10, 2014), https://www.facinghistory.org/resource-library/facing-ferguson-news-literacy-digital-age/officer-kills-ferguson-teen [https://perma.cc/VQY5-P63G].

keys based on pictures and police responding to a call in the real world. Plus, there are plenty of reasons completely unrelated to attack/withdraw research to think that training police officers to quickly categorize suspects might be a bad idea.[304] Nevertheless, given the evidence to date, more research is called for. It is possible (and consistent with quantum principles) that similar effects could be achieved by having people make a preliminary decision other than good guy/bad guy categorization.

Prediction #8: Certain debiasing methods may help reduce the effects of heuristics and biases across the board. From a quantum perspective, heuristics and biases are products of people's use of *incompatible* mental representations.[305] This suggests that debiasing decision makers boils down to getting them to use *compatible* representations. But what affects whether decision makers use compatible, rather than incompatible, representations?

Two recent studies have found that people are more likely to represent events as compatible as they gain familiarity with the relevant task.[306] These findings may provide guidance concerning the contexts in which debiasing policies might be beneficial— heuristics and biases may not be as prevalent (and, thus, debiasing may not be as useful) for tasks that are familiar and well-rehearsed to the relevant decision maker(s). These findings also imply that, where debiasing is needed, familiarization (e.g., training and practice for decision makers) may be a key strategy. For example, quantum decisionmaking lends support to calls for jurors to receive some form of standardized training in logical reasoning.[307]

Another recent study[308] found a significant relationship between participants' individual styles of mental representations (compatible versus incompatible) and a simple cognitive measure called the Cognitive Reflection Test.[309] People with higher scores on the Cognitive Re-

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304. *See, e.g.*, David A. Wilder, *Social Categorization: Implications for Creation and Reduction of Intergroup Bias*, 19 ADVANCES IN EXPERIMENTAL SOC. PSYCHOL. 291 (1986) (discussing formation ingroup/outgroup classifications and their effects).

305. *See, e.g.*, Bruza, Wang & Busemeyer, *supra* note 8, at 392 ("Our view is that incompatibility of events provides an effective solution to bounded resources, which is the reason for bounded rationality.").

306. James M. Yearsley, Jennifer S. Trueblood & Emmanuel M. Pothos. *When Are Representations of Causal Events Quantum Versus Classical?*, *in* PROCEEDINGS OF THE 38TH ANNUAL CONFERENCE OF THE COGNITIVE SCIENCE SOCIETY 2447 (2016), http://mindmodeling.org/cogsci2016/papers/0423/paper0423.pdf [https://perma.cc/6YPW-SRPA]; Trueblood, Yearsley & Pothos, *supra* note 166.

307. *See, e.g.*, Jonathan Koehler, *Train Our Jurors*, *in* HEURISTICS AND THE LAW 303-26 (G. Gigerenzer & C. Engel, eds., 2006).

308. Yearsley, Trueblood & Pothos, *supra* note 306.

309. Shane Frederick, *Cognitive Reflection and Decision Making*, 19 J. ECON. PERSP. 25 (2005). The Cognitive Reflection Test measures an individual's ability to suppress an initial

flection Test were found to be more likely to represent events as compatible than people with lower scores.[310] Again, this may have potential implications for where debiasing efforts should be focused. One may expect, for instance, that judges perform relatively higher on the Cognitive Reflection Test than jurors (though, to our knowledge, this has not been empirically tested). Further, to the extent conveniently-administrable training techniques are shown to increase cognitive reflection, they may be particularly useful components of debiasing programs.

To summarize Part IV, there are several reasons to believe that quantum decisionmaking is relevant to legal thinkers. Perhaps the most fundamental is that policy decisions are routinely based, in part, on models of decisionmaking,[311] and there is mounting evidence that quantum models reflect decisionmaking better than rational choice models.[312] Because quantum decisionmaking provides a principled means of grouping together many of the heuristics and biases on which legal scholars have focused, it addresses (or at least begins to address) the criticism that BLE is ad hoc.[313] Quantum decisionmaking also helps researchers predict new patterns in heuristics and biases and new situations in which people's choices are prone to cognitive quirks or surprising contextual influences.[314] Quantum decisionmaking remains young, but it lays a promising foundation for a systematic program of behavioral research that can contribute substantially to law and policy.

V. LIMITATIONS

While quantum decisionmaking has much to offer, it is not without limitations, real and perceived. First, a common initial response to quantum decisionmaking is that it seems complex—that, without a background in mathematics or physics, quantum decisionmaking is less intuitive than rational choice theory. To this, we reply first that the perceived intuitiveness of rational choice theory is likely due to familiarity with its general maxims (e.g., "people maximize expected utility"). But, at the level of specifics, using those maxims to build tractable mathematical models that predict particular decisions can be

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"gut" response that is incorrect in favor of a deliberative correct response. We note that a natural hypothesis worth investigating is that trained lawyers and judges may tend to score higher on the Cognitive Reflection Test than the population at large.

310. Trueblood, Yearsley & Pothos, *supra* note 166.

311. *See, e.g.*, Jones, *supra* note 4, at 481 ("Law is fundamentally a consumer of behavioral models.").

312. *See, e.g.*, Trueblood & Busemeyer, *supra* note 28; Yearsley & Pothos, *supra* note 35.

313. *See supra* Part IV.A.

314. *See supra* Part IV.B.

quite difficult.[315] Most give little thought to the assumption (embedded in rational choice theory) that people make probability judgments in accordance with classical probability theory, or to the complex set of rules that classical probability theory prescribes. There is no particularly deep reason to think humans make judgments in accordance with those rules, though researchers have long assumed they do. For law to be the best it can be, the behavioral models on which it is premised should be the best available models.[316] Whether quantum decisionmaking provides more accurate models of decisionmaking than rational choice theory is, ultimately, an empirical question, and a growing body of evidence favors the quantum approach.[317]

Second, a critic might contend that, because practitioners of quantum decisionmaking avoid making claims about the biological processes underlying decisionmaking, these researchers are not actually *explaining* the heuristics and biases that they model.[318] Put differently, the critic might say that quantum decision makers represent the atheoretical list of heuristics and biases using an atheoretical set of modeling rules. But this critique could be levied against any computational approach to decisionmaking, including rational choice theory itself. Rational choice theory, after all, says nothing about the specific mental or biological processes involved in decisionmaking—like quantum decisionmaking, it offers an algorithm[319] for predicting the outcomes of people's decisions. Thus, quantum decisionmaking is no more atheoretical than rational choice theory. Further, the rules of quantum decisionmaking reflect an articulable insight into human decisionmaking: human probability judgments are sensitive to context.[320] The growing desire to move beyond this level of explanation and reduce

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315. *See* YATES, *supra* note 110, at 241-303 (discussing how rational choice models of decisionmaking are implemented).

316. *See* Jones, *supra* note 175, at 285 ("[T]he effectiveness of various legal approaches to regulating behavior depends on the validity of the behavioral models on which law grounds these approaches . . . . An inaccurate or only partially accurate behavioral model makes for a squishy, sponge-like fulcrum, which provides law with only inefficient leverage against the human behavior it seeks to affect.").

317. *See, e.g.*, WENDT, *supra* note 25, at 4 (noting the "growing experimental evidence that long-standing anomalies of human behavior can be predicted by 'quantum decision theory.' ").

318. *See generally* Owen D. Jones, *supra* note 5, at 486 ("[Y]ou can't provide a satisfying causal explanation for a behavior by merely positing that it is caused by some psychological force that operates to cause it.").

319. We note, however, that some researchers in quantum decisionmaking suggest that quantum models "do reflect in some way the process of arriving at a given decision," rather than representing pure algorithms. Yearsley & Busemeyer, *supra* note 102, at 100.

320. *E.g.*, Jan Broekaert et al., *Quantum-like Dynamics Applied to Cognition: A Consideration of Available Options*, 375 PHIL. TRANSACTIONS OF THE ROYAL SOC. A - MATHEMATICAL, PHYSICAL, & ENGINEERING SCI., at 1, 2 (2017) ("Quantum probability theory is a [useful] framework for probabilistic assigning [that is] sensitive to context."); Busemeyer et al., *supra* note 26.

decisionmaking phenomena to their biological underpinnings is a relatively recent intellectual development. Research on the biology of decisionmaking is highly valuable, but the term "theory" does not exclusively mean "biological theory."

A critic might worry that quantum decisionmaking is an unfalsifiable theory.[321] Given that a quantum framework can accommodate both rational decisions and irrational decisions, this is an understandable concern. But one valuable feature of the quantum approach is that quantum models can be empirically tested against conventional rational choice models of decisionmaking[322] and against Bayesian models of probabilistic judgments.[323] Indeed, in some contexts (especially contexts where decision makers are familiar with the relevant task) classical models have outperformed quantum models.[324] Thus, while the empirical evidence to date generally favors quantum models, it is not because quantum models are unfalsifiable.

A critic might also argue that, because at this time quantum decisionmaking has only been shown to account for a subset of the dozens of documented heuristics and biases, it is not yet clear whether it is really up to the task of providing a "general theory" for BLE. This is a fair critique. At this stage, the critical point is that real potential is there. Quantum decisionmaking has accounted for each heuristic to which it has been applied, and to our knowledge, it is the first mathematically-coherent theory that can make this claim.[325] Future research should seek to apply quantum models to more heuristics and biases in a wider variety of decisionmaking contexts, comparing the performance of quantum models to alternative models in an ongoing effort to improve our scientific understanding of human decisionmaking.[326] Even if quantum decisionmaking does not ultimately provide a "unified theory" of human decision, it seems likely that its core insight—that humans might judge probabilities in accordance with a coherent,

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321. Philosopher Karl Popper argued that, in order for theories to be "scientific," they must be falsifiable—it must be possible for them to be disproved by empirical evidence. Notably, "[i]n *Daubert v. Merrell Dow Pharmaceuticals*, the Supreme Court identified the 'falsifiability' criterion . . . as the touchstone of what separates science from metaphysics." Sean O'Connor, *The Supreme Court's Philosophy of Science: Will the Real Karl Popper Please Stand Up?*, 35 JURIMETRICS 263, 263 (1995) (citing KARL POPPER, CONJECTURES AND REFUTATIONS: THE GROWTH OF SCIENTIFIC KNOWLEDGE 37 (5th ed. 1989)).

322. *See, e.g.*, Diederik Aerts, Emmanuel Haven & Sandro Sozzo, *A Proposal to Extend Expected Utility in a Quantum Probabilistic Framework*, 2017 ECON. THEORY 1079 (2017).

323. *See* Trueblood, Yearsley & Pothos, *supra* note 166 (Experiment 3 is an example of a situation in which the classical, Bayesian model was a better fit for the data than the quantum model).

324. *E.g.*, *id.*

325. *See supra* Part IV.A and articles cited therein.

326. *See* Tribe, *supra* note 27, at 2 ("The better vision of science is as a continual and, above all, critical exploration of fruitful insights . . . Science is not so much about proving as it is about *im*proving.").

but non-classical, probability theory—represents a critical step on the way to such a "unified theory."

Of course, even if quantum decisionmaking *does* ultimately prove to be a "unified theory" that accounts for all heuristics and biases, that does not necessarily mean that it will prove as valuable to the law as rational choice theory. Perhaps the greatest strength of rational choice theory is the variety of situations in which it generates clear *a priori* predictions.[327] While quantum decisionmaking also generates *a priori* predictions[328], in many situations making clear predictions requires answering several questions: Will the decision maker use compatible or incompatible representations? And, if the answer is incompatible, in what order will the decision maker make the relevant judgments? For quantum decisionmaking to be as broadly tractable as rational choice theory, researchers must develop some concrete guidance for answering these questions.[329] Without a systematic way to predict the answers, quantum decisionmaking is arguably vulnerable to some of the same criticism that has been levied at BLE[330] (and at the heuristics and biases research program more generally).[331]

With respect to the first question—will decision makers use compatible or incompatible representations?—some guidelines may be coming into focus. First, as mentioned above, incompatibility generally provides a safe default assumption.[332] Incompatibility fundamentally reflects limitations on our cognitive resources and has been described as "ubiquitous" in human decisionmaking.[333] Further, the experiential analog of incompatibility can offer some guidance: if it feels difficult to

327. *See* Robert J. MacCoun, *The Relativity of Judgment as a Challenge for Behavioral Law and Economics* (U.C. Berkeley: Center for the Study of Law and Society Faculty Working Papers 2006), https://escholarship.org/uc/item/5v62r8nm [https://perma.cc/R78J-8ADB] (positing that, in the absence of an overarching theory of behavioral economics, "legal scholars may have to accept two competing frameworks for analyzing judgment and choice: A rational economic framework with clear predictions but shaky foundations, and a psychological framework with strong empirical foundations but uncertain *a priori* implications for a given situation."); *Wright & Ginsburg*, *supra* note 8, at 1040 (observing that "rationality is a simplifying assumption made to render modeling of economic interactions among firms and consumers tractable and to harness the powerful mathematical tools of optimization," and that "if behavioral economics is to outperform price theory, its superiority must be proven by its greater predictive power, not merely by the assertion that its underlying assumptions are more 'realistic.' ").

328. *See, e.g.*, Yearsley & Pothos, *supra* note 35 (predicting, *a priori*, an analog to the quantum Zeno effect in human decisionmaking); Wang et al., *supra* note 21 (predicting, *a priori*, a quantum pattern in order effects); Yearsley & Trueblood, *supra* note 22, (findings were consistent with a constraint on order effects derived, *a priori*, from quantum principles).

329. *See generally* MacCoun, *supra* note 327.

330. *See supra* notes 15-16.

331. Gigerenzer, *supra* note 73 (criticizing the heuristics and biases research agenda for lack of a general theory).

332. *See* Bruza, Wang & Busemeyer, *supra* note 8, at 392.

333. Bruza, Wang & Busemeyer, *supra* note 166.

mentally consider the relevant judgments at the same time, they are likely incompatible.[334] This is a useful rule of thumb, but researchers are working on more formal criteria.[335] As discussed in Part IV.B.7, *supra*, these criteria include task familiarity (people's behavior becomes more classically rational as they gain familiarity with a task)[336] and the individual trait of cognitive reflectiveness (people with higher scores on the Cognitive Reflection Test exhibit more classically rational behavior).[337] Thus, while there is not yet a clear-cut algorithm for predicting whether people will use compatible or incompatible representations, there are several factors that can inform such predictions. These factors are often enough to generate predictions in legal situations. For instance, we would predict that, in trials involving multiple claims, jurors—who typically lack experience with the task of evaluating legal claims—will generally rely on incompatible representations to evaluate each claim sequentially.

With respect to the second question—in what order will decision makers make judgments?—we observe that there are a number of legal situations in which the order of judgments is predictable based on the order of questions posed to decision makers. For example, when jurors fill out verdict forms, or citizens vote in elections, or employees fill out human resources forms, we expect that they tend to make judgments in the prompted order. These situations lend themselves to quantum modeling.[338] But there are, of course, also many situations in which the order of judgments is less predictable. Returning one last time to the example of the plaintiff evaluating a settlement offer, the plaintiff might consider the trademark claim before the defamation claim or the defamation claim before the trademark claim. Perhaps future research can identify general principles that predict how people sequence judgments in such situations (analogous to the principles courts apply in sequencing their consideration of sources of law).[339] For example, it may be that litigants, or lawyers talking through settlement options with litigants, tend to consider the highest-dollar claims first. Such principles, if identified, could enable quantum models to

334. *See* Bruza, Wang & Busemeyer, *supra* note 8, at 385 ("Placed in a psychological context . . . a person's understanding of two events, such as two different politicians or two different perspectives on a matter, requires changing from one point of view to another, and the two points of view can imply incompatibility.").

335. Trueblood & Busemeyer, *supra* note 33, at 1548 ("Deciding when two events should be treated as compatible or incompatible is an important research question. There has been some work on this problem for questions involving human judgments.").

336. *See* Yearsley, Trueblood & Pothos, *supra* note 306.

337. Frederick, *supra* note 309.

338. *See, e.g.*, Moore, *supra* note 265.

339. *See* Samaha, *supra* note 284.

generate *a priori* predictions about decisions in a broader variety of situations.

VI. CONCLUSION

Heuristics and biases have long posed a challenging puzzle for rational choice theory and, consequently, for law. Quantum decisionmaking offers a promising new perspective on this puzzle.[340] Quantum decisionmaking offers a new, comprehensive set of rules for describing and predicting people's judgments and decisions—rules that account for "rational" decisions and heuristics and biases alike. These new rules have substantial implications for law and policy.

First, quantum decisionmaking illuminates connections among many of the heuristics and biases that animate BLE—something that BLE itself has struggled to do. Quantum decisionmaking suggests that at least some heuristics and biases are actually the natural by-products of a coherent set of decision-making rules.[341] It also suggests that at least some heuristics and biases share a common root—that they are different manifestations of people "thinking quantum" (i.e., using incompatible representations). Debiasing policies focused on that common root might prove useful in combating a wide variety of "irrational" decisions.

Quantum decisionmaking also generates an array of specific hypotheses that are relevant to law and policy.[342] These include hypotheses about previously-undocumented cognitive quirks[343] and interesting contextual influences[344] with implications ranging from the courtroom to police training. In sum, quantum decisionmaking offers a useful new perspective on how people make legally-relevant decisions—a perspective that can inform future legal research and perhaps, ultimately, policy.

VII. TECHNICAL APPENDIX

In this Technical Appendix, we demonstrate how to construct the quantum model used in Part III.B of the Article, explain how to "read"

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340. *See generally* Tribe, *supra* note 27, at 2 ("[T]he metaphors and intuitions that guide physicists can enrich our comprehension of social and legal issues.").

341. *See* Bruza, Wang & Busemeyer, *supra* note 8, at 387 ("Rather than resorting to heuristics, quantum cognition successfully accounts for these violations using a coherent, common set of principles.").

342. *See supra* Part IV.B.

343. For example, the "quantum Zeno effect." *See* Yearsley & Pothos, *supra* note 35; *supra* Part IV.B (prediction #1).

344. For example, the influence of categorizing on deciding. *See* Wang & Busemeyer, *supra* note 294; *supra* Part IV.B (prediction #6).

the information it provides, and contrast it with the classical probability framework assumed by rational choice theory.

Quantum models of judgment(s) are built using the rules of quantum probability theory.[345] Quantum probability theory, in turn, is grounded in geometry.[346] Thus, the heart of constructing quantum models is geometrically representing the relevant judgment(s).

A quantum model might involve a single "random variable"[347] (i.e., an isolated judgment like, "did the defendant commit murder?"). It might also involve multiple random variables (i.e., a set of two or more judgments, such as "did the defendant commit murder?" and "is the defendant a convicted felon?"). We illustrate both types of models below, and the difference is crucial. In the context of a single random variable, the quantum approach will produce the same results as the classical approach.[348] But, in the context of multiple random variables, quantum and classical approaches can yield different results.[349] This discrepancy is what allows quantum models to capture contextual influences on judgment.[350] And this sensitivity to context is what enables quantum decisionmaking to account for, and predict, heuristics and biases that are difficult to reconcile with rational choice models.[351]

A. *Modeling a Single Random Variable*

To begin, we will model a situation involving a single random variable. Imagine a juror in a murder trial. Her decision (i.e., her verdict) will be informed by her assessment of the probability that the defendant committed murder. Let the random variable M represent the question "Did the defendant commit murder?" with two possible answers: "true" (which we will abbreviate as "m_t") and "false" (which we will abbreviate as "m_f").

345. Yearsley & Busemeyer, *supra* note 100, at 100 ("Quantum cognition is a framework for constructing cognitive models based on the mathematics of quantum probability theory.").

346. *See* Busemeyer & Wang, *supra* note 23, at 167 ("Classical probability theory is built upon set theory, while quantum probability theory (i.e., the mathematical foundation of quantum theory) is built upon geometric theory.").

347. When a decision maker makes a decision about an unknown, it is customary in probability theory to refer to the unknown as a "random variable." HWANG & BLITZSTEIN, *supra* note 115, at 91-94.

348. *See, e.g.*, Pothos & Busemeyer, *supra* note 35 (noting that "the predictions between [classical probability] theory and quantum probability] theory with compatible questions would be identical.").

349. *See* Bruza, Wang & Busemeyer, *supra* note 8, at 384-85.

350. Zheng Wang et al., *The Potential of Using Quantum Theory to Build Models of Cognition*, 5 Topics in Cognitive Sci. 672, 674 (2013) (noting that, in order to capture order effects in judgment, "we cannot define a joint probability of answers simultaneously to a conjunction of questions" but instead "can only assign a probability to the sequence of answers.")

351. Busemeyer et al., *supra* note 26.

In classical probability theory, we would represent this situation with a sample space containing two events: m_t (the defendant committed murder) and m_f (the defendant did not commit murder).[352] A Venn diagram of our sample space would contain a region that corresponds to the event m_t and a region that corresponds to the event m_f, as illustrated in Figure A1. The juror's assessment of the probability that the defendant committed murder is reflected by the size of the m_t region relative to the size of the whole sample space. The bigger the m_t region, the higher the probability the juror assigns to the event that the defendant committed the murder. Assume that, in Figure A1, the m_t region fills 16% of the sample space. Thus, the juror believes there is a 16% chance (or 0.16 probability) that the defendant committed the murder.

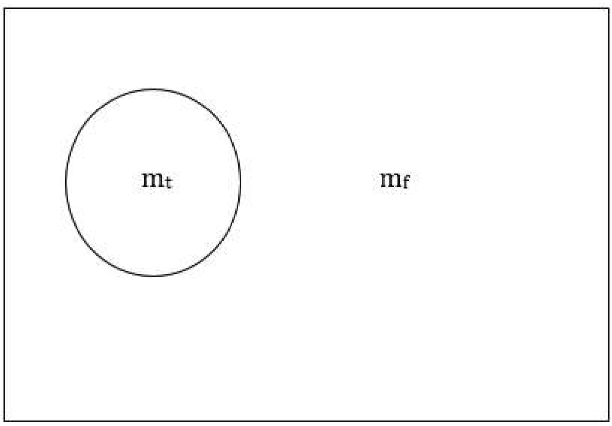

Figure A1. Sample space for juror's judgment. The square represents the sample space. The circular region labeled "m_t" represents the event that the defendant committed murder. The remaining region labeled "m_f" represents the event that the defendant did not commit murder.

Alternatively, we can represent the same situation geometrically in a quantum model. We will walk through the steps of constructing this model.

First, in a quantum model, we associate the two events m_t and m_f with two different vectors. In geometric terms, a "vector" is a line with a specified direction (meaning it starts at Point A and ends at Point B).[353] Because m_t and m_f are complementary events (they both cannot

352. *E.g.*, YATES, *supra* note 110, at 114-15.

353. *Vector*, THE CONCISE OXFORD DICTIONARY OF MATHEMATICS (5th ed. 2014).

simultaneously be true), we define the associated vectors to be orthogonal (i.e., perpendicular) to one another. You can think of them like axes providing the coordinate system.[354] These two vectors provide the foundation of our geometric quantum model. Technically, these vectors are called "basis vectors" and define a geometric space called a "vector space".[355]

Because we are interested in probabilities, we need to work with numbers between 0 and 1. To accomplish this, we restrict our model to a "unit circle." A unit circle is a circle that has a radius of 1, such that every point on the outside edge of the circle is exactly 1 unit away from the center of the circle.[356] The left-hand side of Figure A2 depicts our unit-circle model containing m_t and m_f basis vectors.

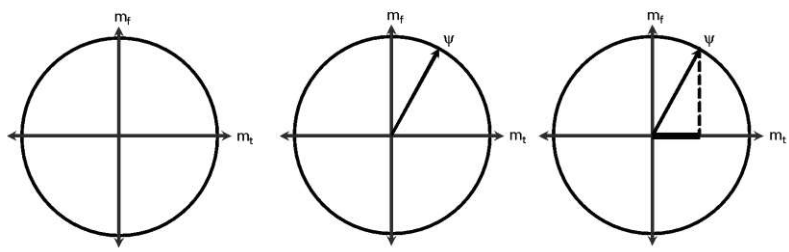

Figure A2. Constructing a quantum model of a single juror judgment (i.e., one random variable). The left panel illustrates a 2-dimensional unit circle with basis vectors m_f ("the defendant did not commit murder") and m_t ("the defendant committed murder"). In the middle panel, we add the belief state of the decision maker as a new vector in our space, labeled ψ. In the right panel, we calculate the probability the decision maker assigns to the event m_t based on the decision maker's current belief state. Mathematically, the probability is calculated by projecting the belief state onto the m_t basis vector.

354. *See* BUSEMEYER & BRUZA, *supra* note 21, at 31 ("The basis vectors corresponding to the elementary outcomes are orthogonal; that is, at right angles to each other."); *see also* Bruza, Wang & Busemeyer *supra* note 8, at Figure 3 (providing an illustration).

355. *See* Wang et al., *supra* note 21, at 9433-34 (defining "basis vectors" and illustrating a "vector space.").

356. *See, e.g.*, Keith Weber, *Teaching Trigometric Functions: Lessons Learned from Research*, 102 MATHEMATICS TEACHER 144, 148 ("A unit circle is a circle with a radius of 1 whose center is the origin," with the term "origin" referring to the point (0,0) in a Cartesian plane); *see also Origin*, MATH OPEN REFERENCE, https://www.mathopenref.com/origin.html [https://perma.cc/3EYP-NZX3] (the origin is the point where the axes intersect; in a two-dimensional plane, "[t]his point has the coordinates 0,0 and is usually labeled with the letter O.").

In addition to basis vectors, our quantum model needs a "belief state," representing what the decision maker is currently thinking.[357] By convention, the belief state is labeled using the Greek letter psi, ψ.[358] The belief state is a vector that has a length of exactly one unit.[359] On a general level, you can think of this vector as a needle pointing toward the decision maker's beliefs. The belief state vector always originates in the center of the unit circle and always ends somewhere on the outer edge of the unit circle.[360] Where, precisely, the vector ends tells us something about the decision maker's beliefs.[361] In general terms, the closer the belief state is to a certain basis vector, the higher the likelihood the decision maker assigns to the event represented by that basis vector. In the middle panel of Figure A2, we have added a belief state, labeled ψ, to our model.

With the basis vectors and the belief state now defined, we can compute the probability that the juror assigns to a particular event. To do so, we use a geometric operation called "projection."[362] Specifically, we "project" the belief state onto the basis vector corresponding to the event of interest. [363] The probability that the juror assigns to that event is equal to the squared length of the resulting projected vector.[364]

This computation is not as daunting as it may sound in the abstract. Assume we want to know the probability the juror assigns to the event m_t (the event that the defendant committed the murder). We need to "project" the juror's belief state onto the m_t basis vector. Functionally, all this means is that we draw a straight line from the end of the belief

357. Yearsley & Busemeyer, *supra* note 100, at 101 ("The second ingredient in any quantum model is the specification of the initial knowledge state of a participant, or group of participants," which is captured in quantum models by the belief state); Trueblood & Busemeyer, *supra* note 31, at 1522-1523 (discussing how quantum models represent belief states with a vector).

358. Yearsley & Busemeyer, *supra* note 100, at 102. Note that some sources represent the vector corresponding to the belief state with the letter "S." *E.g.*, Bruza, Wang & Busemeyer, *supra* note 8, at Figure 3.

359. Trueblood & Busemeyer, *supra* note 31, at 1523 (belief states are defined as unit length to ensure that the probabilities it will be used to calculate are between 0 and 1).

360. *Id.*

361. Importantly, however, so long as the belief state is not perfectly aligned with one of the basis vectors, the decision maker has not yet made up her mind for certain. Absent perfect alignment, the belief state is said to represent a "superposition"—a situation where, for example, the juror simultaneously believes both "the defendant committed the murder" and "the defendant did not commit the murder." Psychologically, the juror is in an ambiguous state with respect to whether the defendant committed the murder. Busemeyer & Trueblood, *supra* note 33, at 2.

362. Bruza, Wang & Busemeyer, *supra* note 8. For a detailed discussion of projection, *see* BUSEMEYER & BRUZA, *supra* note 21.

363. BUSEMEYER & BRUZA, *supra* note 21; *see also* Trueblood & Busemeyer, *supra* note 31, at 1523; Busemeyer et al., *supra* note 27, at 195.

364. BUSEMEYER & BRUZA, *supra* note 21, at 31; Bruza, Wang & Busemeyer, *supra* note 8, at 386.

state to the m_t basis vector, such that the line intersects the m_t basis vector at a right angle.[365] In the right-hand side of Figure A2, the projection is represented by the dotted line. This process results in a "projected vector," which is a vector that extends from the center of our unit circle to the point where the projection (the dotted line) intersects with the basis vector. In the right-hand side of Figure A2, the thick black line represents the length of the projected vector (assume it is 0.4). Now, to find the probability that the juror assigns to the event m_t, we square the length of that thick black line.[366]

Note that we could ask people (i.e., research participants) to make judgments analogous to the one that the juror is making in our example. We could then adjust the position of the belief state in our model based on our observations, just as we could adjust the size of the m_t region in a Venn diagram of a classical sample space.[367] Moving the belief state closer to the m_t basis vector in Figure A2 would be analogous to increasing the size of the m_t region of a classical sample space—in general terms, both would reflect an increase in the decision maker's assessment of the probability that that the defendant committed murder.

In the quantum model that we depict in Figure A2, the belief state is positioned so that the squared length of the m_t projected vector is 0.16: the juror believes there is a 16% chance (or 0.16 probability) that the defendant committed the murder. Thus, this quantum model depicts precisely the same judgment that was depicted classically in the Venn diagram in Figure A1 above.

At this point, the reader might ask why we would bother with all the geometry in order to model a judgment that we could (and did) model more easily using a classical approach. The answer is that, when we deal with more than one random variable (e.g., multiple judgments), important differences between quantum probability models and classical probability models emerge.

B. Modeling Multiple Random Variables

In the previous section, we walked through how one can represent and calculate probabilities associated with a single random variable in

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365. BUSEMEYER & BRUZA, *supra* note 21, at 15-16, 31. One helpful analogy is that "[p]rojection is akin to shining a light from above and seeing the length of the shadow onto the plane." Bruza, Wang & Busemeyer, *supra* note 8, at 386.

366. *See* BUSEMEYER & BRUZA, *supra* note 21, at 31 ("The [belief] state is projected onto the [basis vector] corresponding to an event, and the squared length of this projection equals the event probability.").

367. *See* BUSEMEYER & DIEDERICH, *supra* note 101, at 7 ("Model development and testing is a never-ending process," as models are modified, extended, or otherwise reconstructed to account for experimental results.).

a quantum model. Importantly, with respect to a single random variable, the quantum model does not tell us anything different than a classical probability model.[368] It is simply a different way to think about the same problem—a different road leading to the same place.

But the critical differences between quantum probability theory and classical probability theory arise when a situation involves the evaluation of multiple random variables.[369] In this case, quantum probability theory—unlike classical probability theory—is highly sensitive to *context*. That is, quantum theory allows for the possibility that, in some cases, processing one random variable (i.e., making one judgment) can systematically affect the way subsequent random variables are processed (i.e., subsequent judgments are made).[370]

For example, suppose the juror in our hypothetical murder trial is also considering the possibility that the defendant was previously convicted of another felony. Thus, in addition to the random variable M that relates to the juror's murder judgment, let the random variable C represent the question "Is the defendant a convicted felon?" with two possible answers: "true" (which we will abbreviate as "c_t") and "false" (which we will abbreviate as "c_f").

To model the random variables M and C using a classical probability approach, we have to redefine our sample space to reflect four possible combinations of choices: the defendant both committed murder and is a convicted felon (m_t & c_t), the defendant committed murder but is not a convicted felon (m_t & c_f), the defendant did not commit murder but is a convicted felon (m_f & c_t), and the defendant did not commit murder and is not a convicted felon (m_f & c_f).[371] (We must do this because, as the reader may recall, the sample space is exhaustive in classical probability theory—it contains all possible events.[372]) A Venn diagram of our sample space would now contain four regions instead of two, as shown in Figure A3.

368. *See* Bruza, Wang & Busemeyer, *supra* note 8, at 391 ("The need for the quantum approach only arises when incompatible events are involved.").

369. *Id.*

370. Trueblood & Busemeyer, *supra* note 31, at 1524-26; Bruza, Wang & Busemeyer, *supra* note 8, at 383-84; Yearsley & Busemeyer, *supra* note 100, at 107.

371. *See* Bruza, Wang & Busemeyer, *supra* note 8, at 385 (explaining that in classical probability theory, a universal sample space contains each event and its intersection with other events).

372. *See supra* Part III.A.3.

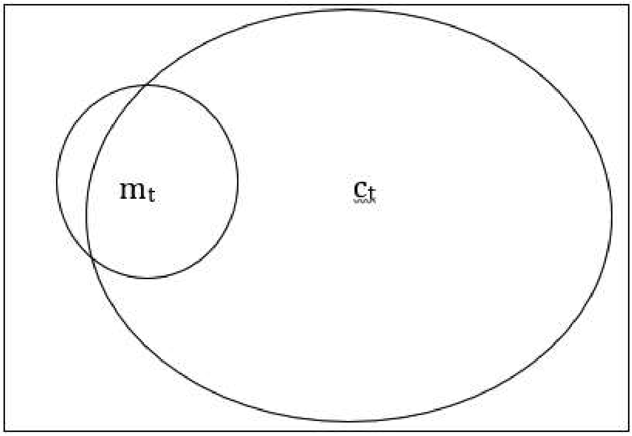

Figure A3. Sample space for juror's judgments. The square represents the sample space, and the circles represent the event that the defendant committed murder (m_t) and the event that the juror is a convicted felon (c_t). The overlap of the two circles represents the joint event in which the defendant both committed murder and is a convicted felon (m_t & c_t). The area outside of the circles represents the joint event in which the defendant did not commit murder and is not a convicted felon (m_f & c_f).

To construct a quantum model of the juror's judgments, we must first make a choice about the relationship between them. Specifically, the M and C variables can either be "*compatible*" or "*incompatible*"—and our choice will dictate the form that our quantum model takes.[373] Variables are "compatible" if they commute, meaning that the order in which they are processed does not matter.[374] In other words, if events are compatible, thinking about one thing (e.g., one judgment) will not influence thoughts about the second thing (e.g., a second judgment)—the decision makers' judgments are not sensitive to context. When variables are compatible, the quantum model will be functionally equivalent to the classical probability model.[375]

373. Bruza, Wang & Busemeyer, *supra* note 8, at 384.

374. *Id.* ("[T]wo questions are compatible if they can be answered simultaneously, or even if they are answered sequentially, the order does not matter; two questions are incompatible if they have to be asked sequentially and the order does matter.").

375. *See* Busemeyer et al., *supra* note 27, at 198 ("If all events are compatible, then quantum theory is equivalent to classic theory.") (citing STANLEY P. GUDDER, QUANTUM PROBABILITY (1988)). The geometric details of quantum models of compatible events are beyond the scope of this Article. For our purposes, it is enough to know that compatible models do not produce different results than classical models.

However, we might believe that thinking about someone being a convicted felon would influence thoughts about them also being a murderer. That is, we may suspect that the juror's processing one of the variables (i.e., making one judgment) in the model will interfere with the juror's processing the other variable (i.e., making the other judgment). This intuition is captured in a quantum model by using "incompatible" variables.[376]

When two variables are incompatible, we use *two different sets* of basis vectors to represent the events associated with them.[377] In our example, if we think M and C are incompatible, we can use one pair of basis vectors that relates only to the murder judgment (basis vectors m_t and m_f), and another pair of basis vectors that relates only to the convicted felon judgment (basis vectors c_t and c_f). The second set of basis vectors are represented in our model as a rotation of the first set.[378] More rotation generally indicates a stronger contextual influence of one judgment on the next.[379] In the left-hand side of Figure A4, the basis vectors for c_t and c_f appear in light gray.

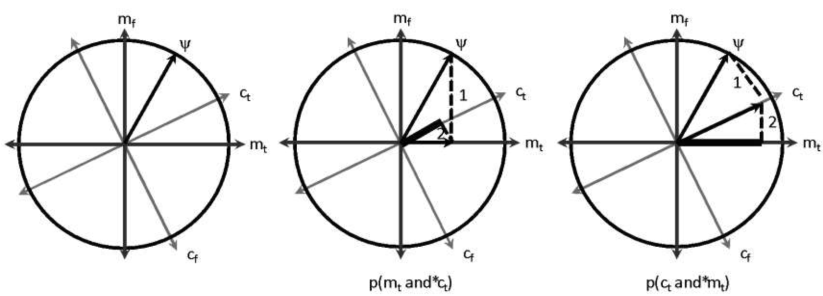

Figure A4. Constructing a quantum model of two juror judgments (i.e., two random variables). The left panel illustrates the two pairs of basis vectors, one pair for the random variable M (m_f = the defendant did not commit murder; m_t = the defendant did commit murder) and

376. *E.g.*, Bruza, Wang & Busemeyer, *supra* note 8, at 385; Wang et al., *supra* note 21, at 9435; Busemeyer & Trueblood, *supra* note 33, at 5.

377. Trueblood & Busemeyer, *supra* note 31, at 1524-25. Note that it is possible to define infinitely many sets of basis vectors within a vector space. Thus, it is always possible to define a new set.

378. To determine how much we rotate the second set of basis vectors from the first, we would collect some data to "fit" our model. For a description of this process, see, for example, Busemeyer et al., *supra* note 27, at 209.

379. *See* Bruza, Wang & Busemeyer, *supra* note 8, at 386 ("The degree of rotation between the two [sets of basis vectors] is determined by the similarity between [perspectives] (i.e., less rotation means the perspectives are more similar."); *see also* Yearsley & Busemeyer, *supra* note 100, at 107 ("[T]he degree of incompatibility, measured through non-commutation or order effects, depends on the overall size of the unitary rotation between bases . . . .").

one pair for the random variable C (c_f = the defendant is not a convicted felon; c_t = the defendant is a convicted felon). The middle panel illustrates the projection sequence for judging the probability that the defendant committed murder (m_t) followed by judging the probability that the defendant is a previously-convicted felon (c_t). The right panel illustrates the reverse order of judgments.

To calculate the probabilities associated with incompatible events, we apply a *sequence* of projections.[380] Let us say that we want to calculate the juror's judgment of the joint probability that the defendant committed murder and is a convicted felon (the event m_t & c_t). We first have to decide the order of the projections.[381] We can project the belief state onto either the m_t basis vector or the c_t basis vector first. Importantly, the order matters.[382]

Figure A4 shows why. In the middle panel, the juror judges the probability that the defendant committed murder first, and the probability that the defendant is a previously-convicted felon second, so we sequence the projections "m_t-c_t." That is, we first project the belief state onto the m_t basis vector, then we take the resulting projected vector and project it onto the c_t basis vector. In the right-hand panel, the juror judges the probability that the defendant is a previously-convicted felon first, and the probability that the defendant is a murderer second, so we sequence the projections "c_t-m_t." That is, we first project the belief state onto the c_t basis vector, then we take the resulting projected vector and project it onto the m_t basis vector.

Critically, the projected vectors that result from the "m_t-c_t" sequence and the "c_t-m_t" sequence *are not equal* (compare the thick black bars in the middle and right panels of Figure A4). This reflects that the context in which the juror makes each judgment (*e.g.*, whether she makes it before or after the other judgment) matters. In this model, the juror assigns a higher probability to the event that the defendant committed murder if she first judges the probability that the defendant is a convicted felon. Our quantum model captures this contextual influence of the "convicted felon" judgment easily. Classical probability models have difficulty with this, because nothing in classical models is naturally sequential (see the "Venn diagram" in Figure A3).[383]

380. Trueblood & Busemeyer, *supra* note 31, at 1518, 1536.

381. BUSEMEYER & BRUZA, *supra* note 21, at 360.

382. *See* Bruza, Wang & Busemeyer, *supra* note 8, at 388-91.

383. *See, e.g.*, Busemeyer et al., *supra* note 27, at 199 (With a classical probability approach, events can be evaluated without regard to order.); YATES, *supra* note 110, at 114-118 (discussing probability estimates in classical probability framework).

Quantum models of incompatible events, like the one depicted in Figure A4, can account for and predict a number of heuristics and biases. For example, if evidence were introduced that proved the defendant is a convicted felon, then the quantum model would naturally predict that the juror's murder verdict would be influenced by representativeness / base rate neglect.[384] Assume that the juror learns, definitively, that the defendant is a convicted felon and is then judging whether the defendant committed murder. Because the juror *knows* the defendant is a convicted felon, the juror is judging the conditional probability that the defendant committed murder *given that* the defendant is a convicted felon. In the language of probability theory, this conditional probability can be expressed as $p(m_t \mid c_t)$ (note that the "|" symbol simply means "given").[385]

Figure A5 shows how we calculate this probability. Since c_t is a given, we project the juror's belief state (labeled ψ) onto the c_t basis vector first (dotted line 1 in the figure). Then, because the juror *knows* the defendant is a convict, we have to "normalize" the projected belief state—this reflects that, from the juror's perspective, $p(c_t) = 1$ (i.e., the likelihood of the suspect being a convicted felon is 100%).[386] Normalization gives us a revised belief state, labeled ψ_{ct} in the figure, which reflects the juror's revised beliefs after having learned, definitively, of the prior conviction. The revised belief state is then projected onto the m_t basis vector (denoted by dotted line 2 in the figure).[387] The conditional probability $p(m_t \mid c_t)$ is the squared length of this final projection (denoted by the solid black bar in Figure A5).[388] As illustrated in Figure A5, when our juror has knowledge that the defendant is a convicted felon, the probability that she assigns to the event that the defendant committed murder (m_t) increases above what the probability she would have assigned if she had simply judged the events c_t and m_t in

384. *See supra* Part III.B.

385. *E.g.*, YATES, *supra* note 110, at 127-29 (introducing and discussing conditional probabilities).

386. In quantum probability theory, normalization of the belief state is part of the process in calculating conditional probabilities. The belief state is normalized whenever it is projected onto the conditioning event (e.g., ct). Conceptually, this is because we know that if a condition is true, its probability is 1. *See* Trueblood & Busemeyer, *supra* note 31, at 1524 (describing how normalization reflects a revision to beliefs after observing an event); Wang et al., *Supporting Information* for *Context Effects Produced by Question Orders Reveal Quantum Nature of Human Judgments*, PROC. NAT'L. ACAD. SCI. 4 (2014), https://www.pnas.org/content/pnas/suppl/2014/06/11/1407756111.DCSupplemental/pnas.201407756SI.pdf [https://perma.cc/D5FK-ST8M] [hereinafter Wang et al., *Supporting Information*] ("Quantum theory assumes that, after a first choice is made, the revised belief state is formed from the initial belief state by projecting the initial belief state onto the [basis vector] for an answer to a question and normalizing the projection to form the revised belief state.").

387. Wang et al., *Supporting Information*, *supra* note 388.

388. BUSEMEYER & BRUZA, *supra* note 21, at 31; Bruza, Wang & Busemeyer, *supra* note 8, at 386.

the same order but in the absence of dispositive evidence of prior conviction (compare Figure A5 to the right-hand panel of Figure A4).[389] Critically, base rates never factored into our conditional probability calculation. Thus, our quantum model reflects precisely the problem that scholars have identified as justification for Federal Rule of Evidence 404.[390]

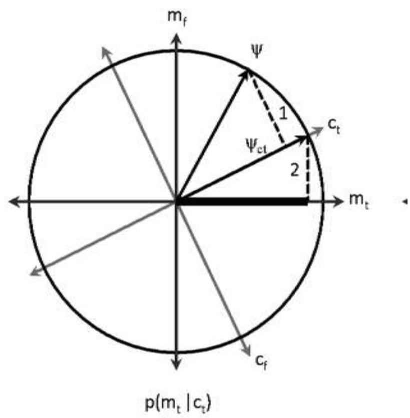

Figure A5. Quantum model illustrating representativeness / base rate neglect in judgment of the probability the defendant committed murder ($p(m_t)$) when the defendant's prior conviction is a given ($p(c_t) = 1$).

Finally, we note that the model in Figure A5 also naturally predicts another observed irrationality called the "inverse fallacy"—the tendency of people to incorrectly equate the conditional probability they are trying to judge with its inverse.[391] Our juror would commit the inverse fallacy if she equated the probability that the defendant is a murderer given that the defendant is a convict ($p(m_t \mid c_t)$) with the probability that the defendant is a convict given that the defendant is a murderer ($p(c_t \mid m_t)$). And, indeed, if you perform the reverse set of calculations on the quantum model to compute ($p(c_t \mid m_t)$), you will find it is (necessarily) equal to $p(m_t \mid c_t)$.[392]

389. This probability will necessarily increase when the probability that the defendant is a convicted felon goes from less than 1 to 1.

390. *See* Korobkin & Ulen, *supra* note 4, at 1087.

391. *See* Eddy, *supra* note 200, at 251-59.

392. Note that this is only a necessity in two-random-variable models.